Base-Station and Subcarrier Assignment in Two-Cell OFDMA Downlink under QoS Fairness

(Invited Paper)

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***Abstract*— Consider the problem of base-station and subcarrier assignment on the downlink of a two-cell OFDMA system with adaptive modulation. The aim is to maximize a common data rate that can be offered to each user with a guaranteed maximum rate-outage probability. A target bit-error rate is also guaranteed with a maximum outage probability. The channel model takes into account the propagation path-loss in addition to log-normal shadowing and Rayleigh fading. The channel-state information available about each user corresponds to the *shadowed path-loss* that we define. We use the well-known frequency-reuse partitioning as an inter-cell interference mitigation technique. We suggest a simple threshold-based method for optimizing the reuse scheme. This method allows us to transform the considered optimization problem into a tractable one that we resolve numerically. Simulation results allows us to validate the proposed method and to compare the average achieved performance to that obtained with static BS-assignment and/or static frequency-reuse scheme.**

I. Introduction

Orthogonal Frequency-Division Multiple Access (OFDMA) has become a widely adopted modulation and multiple access technique for high-speed data applications as in Wimax [1], [2] and 4G systems. OFDMA offers an efficient solution due to its flexibility in terms of resource allocation. An intense research activity [3]–[11] is devoted to resolve different problems of resource allocation in OFDMA based on cross-layer optimization approaches [12]. Some efforts have considered a single cell system while others [8], [9] have focused on the multi-cell case where inter-cell interference is involved .

OFDMA resource allocation problems become more challenging when some Quality-of-Service (QoS) fairness constraints are considered [10], [11] . This usually leads to non-tractable joint optimization problems for which heuristic and suboptimal solutions need to be found. In this paper, we consider the downlink of a two-cell OFDMA system and aim at proposing a simple resource allocation algorithm for fair QoS service provision. Our approach is a suboptimal solution to the problem of base-station and subcarrier assignment under adaptive modulation (see some examples of related work in [13] and [14]). We consider the following QoS fairness constraints: a common data rate is offered to each user subject to a maximum allowed outage probability. Moreover, a target Bit-Error Rate (BER) is guaranteed to all users with a prescribed maximum BER-outage probability. Our aim is to maximize the common data rate under a limited total power per Base Station (BS).

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In order to improve the spectral efficiency of our solution, we use the well-known *frequency-reuse partitioning* scheme as an Inter-Cell Interference (ICI) mitigation technique (See [15] and references therein). This technique consists in dividing the available frequency-band into a “full-reuse sub-band” and a “partial-reuse sub-band”. The full-reuse sub-band is formed by subcarriers that are shared in both cells. In contrast, each subcarrier in the partial-reuse sub-band is exclusively used in one of the two cells. Typically, full-reuse subcarriers are assigned to users who are close to their serving BS thus relatively isolated from the interfering other BS. Edge users who undergo severe ICI prefer to be assigned to interference-free partial-reuse subcarriers. This description takes into account a geographical criterion for reuse-scheme selection when the channel effect is reduced to distance-dependent path-loss [16]. In this study we consider a realistic channel model with random shadowing and fast fading. We assume that both BSs have the knowledge of the local-mean of the channel power-gain of each user. This partial Channel-State Information (CSI), that we call the *shadowed path-loss*, represents the statistic expectation, with respect to the fading process, of the channel power-gain. The shadowed path-loss accounts for both path-loss and shadowing. It can be evaluated in practice by averaging the received power over the different subcarriers and sending back the obtained estimation to the serving BS. Inter-BS cooperation allows an exchange of CSI in addition to other necessary signaling for resource usage coordination.

The remaining of this paper is organized as follows. In section II the system model is described. The considered optimization problem is formulated in section III. Then, the proposed solution to simplify this problem is presented in section IV. Section V provides expressions for the achievable user rate and spectral efficiency. Numerical results are presented and commented in section VI. Finally, some concluding remarks and work perspectives are given in section VII.

II. System and Signal Model

Consider a two-cell system with N_u uniformly-distributed users. Each cell is a circle of radius R centered on its BS. Base stations are at distance $d < 2R$ from each other. Each BS has a total peak power P_{tot} equally-partitioned over N_s subcarriers. The frequency-selective channel between BS b

$(b=1,2)$ and user u is characterized by the random variables $g_{b,u,s}$ $(s=1,...,N_s)$ representing the channel power gains over the different subcarriers. Each coefficient $g_{b,u,s}$ accounts for a path-loss $G(x_{b,u})$, that depends on the distance $x_{b,u}$ of user u to BS b, in addition to a *log-normal shadowing* $10^{\,0.1\,\xi_{b,u}}$ and to a multi-path *squared Rayleigh power-fading* $\phi^2_{b,u,s}$. This can be expressed as follows

$$g_{b,u,s} = G(x_{b,u}) 10^{\,0.1\,\xi_{b,u}}\ \phi^2_{b,u,s}. \tag{1}$$

We assume that the path-loss $G(x_{b,u})$ follows the *exponent model* [17] defined by

$$G(x_{b,u}) = \frac{G_0}{x^{\alpha}_{b,u}} \tag{2}$$

where $\alpha \geq 2$ is the *path-loss exponent* that depends on the terrain nature and on the BS antenna height [17]. The constant G_0 is given by $G_0 = (c/(4\pi f_c))^2$ where f_c is the center frequency and c is the light speed. In (1), the log-normal shadowing $10^{\,0.1\,\xi_{b,u}}$ is characterized by the *shadowing standard deviation* σ which is the standard deviation of the zero-mean Gaussian random variable ξ_u.

We define the *channel shadowed path-loss* between BS b and user u by

$$\overline{g}_{b,u} = G(x_{b,u})\ 10^{\,0.1\,\xi_{b,u}}. \tag{3}$$

This shadowed path-loss follows a log-normal distribution $\mathcal{LN}(10\log_{10} G(x_{b,u}), \sigma^2)$. The pair $(\overline{g}_{1,u}, \overline{g}_{2,u})$ represents the CSI available to both BSs about user u. The shadowed path-loss can be estimated in practice by averaging the received power from each BS over the subcarriers during a dedicated OFDM symbol. This CSI depends on the shadowing realization which is assumed very-slowly varying with respect to the resource allocation refresh period. On the other hand, the fading process varies more quickly compared to the shadowing.

At any time, a user u is assigned to one BS b_u called the "serving BS". The index of the other cell is denoted by $\overline{b}_u$. Thus, the vector $[b_u]$ describes the BS assignment which makes part of the resource allocation scheme. Moreover, each user must also be assigned to one or more subcarriers during the current frame. With reuse-partitioning, some subcarriers are reused in the other cell $\overline{b}_u$. The set of reused subcarriers forms the "full-reuse sub-band". On the contrary, each one of the remaining subcarriers which form the "partial-reuse sub-band" is exclusively used by one of the two BSs.

Assume that user u is assigned to BS b_u and to subcarrier s. Depending on the reuse-factor of subcarrier s, the received signal at user u suffers or not from co-channel interference resulting from the "Interfering BS" $\overline{b}_u$. Thus, the *Signal-to-Interference-plus-Noise Ratio* (SINR) is

$$\gamma_{b_u,u,s} = \begin{cases} \frac{(P_{tot}/N_s)\ g_{b_u,u,s}}{BN_0}, & \text{if } s \text{ is not reused,} \\ \frac{(P_{tot}/N_s)\ g_{b_u,u,s}}{BN_0 + (P_{tot}/N_s)\ g_{\overline{b}_u,u,s}}, & \text{if } s \text{ is reused} \end{cases} \tag{4}$$

where B is the subcarrier spacing and N_0 is the AWGN power spectral density. In the following we suppose that all the subcarriers assigned to a given user of index u have the same reuse-factor denoted by f_u. We have $f_u = 1$ (resp. $f_u = 2$) for reused (resp. non-reused) subcarriers so that (4) can be compacted as follows

$$\gamma_{b_u,u,s} = \frac{\gamma_0\ g_{b_u,u,s}}{1 + (2 - f_u)\gamma_0\ g_{\overline{b}_u,u,s}} \tag{5}$$

with $\gamma_0 = P_{tot}/(N_s B N_0)$.

III. Problem Statement

Our aim is to find the optimal base-station assignment, frequency-reuse scheme, subcarrier and rate allocation that maximizes a common data rate r_c guaranteed to all users with probability at least $1 - \mathcal{P}_r$. This means that the data rate of a given user may be lower than r_c but with probability bounded by $\mathcal{P}_r$. In this case, this user is in rate-outage and, consequently, we call $\mathcal{P}_r$ the maximum rate-outage probability. We also require that when a given user is not in rate-outage a target BER β is guaranteed with probability $1 - \mathcal{P}_\beta$.

Let M_u be the modulation order on subcarriers assigned to user u. Conditionally to the CSI $(\overline{g}_{1,u}, \overline{g}_{2,u})$, the choice of M_u must guarantee the target BER-outage probability given the fading statistics. Let $\beta_{b_u,u,s}$ be the actual BER for user u assigned to BS b_u on subcarrier s. We use the well-known Gap Approximation [18] for uncoded M-QAM BER

$$\beta_{b_u,u,s} = 0.2\exp\left(\frac{-1.6\ \gamma_{b_u,u,s}}{M_u - 1}\right) \tag{6}$$

where $\gamma_{b_u,u,s}$ is the instantaneous SINR defined in (5).

Beside $[b_u]$, $[f_u]$ and $[M_u]$, an additional degree of freedom is the subcarrier allocation scheme. Since the available CSI $(\overline{g}_{1,u}, \overline{g}_{2,u})$ is not a frequency-selective information, the subcarrier allocation is transformed into a bandwidth allocation. For analysis purpose, we consider a continuous bandwidth allocation (as if the number of subcarriers tends to infinity). Let B_u be the bandwidth allocated to user u. Thus, the considered user achieves a data rate

$$r_u = B_u \log_2 M_u. \tag{7}$$

Finally, we define the following quantity

$$B_{b,f} = \sum_{u:\ b_u=b,\ f_u=f} B_u. \tag{8}$$

For example, $B_{1,2}$ represents the bandwidth allocated to those users who are assigned to BS 1 with a reuse-factor two. Thus, the considered optimization problem can be formulated as follows

$$\max_{[b_u],[f_u],[M_u],[B_u]} r_c \quad \text{subject to} \tag{9}$$

$$\begin{cases} \text{[C1]} & B_{1,1} = B_{2,1}, \\ \text{[C2]} & B_{1,1} + B_{1,2} + B_{2,2} = B_{tot}, \\ \text{[C3]} & \Pr\left\{\beta_{b_u,u,s} > \beta \mid (\overline{g}_{1,u}, \overline{g}_{2,u})\right\} = \mathcal{P}_\beta,\ \ \forall u, s, \\ \text{[C4]} & \Pr\{r_u < r_c\} \leq \mathcal{P}_r,\ \ \forall u. \end{cases}$$

The first constraint [C1] states that BSs must agree on the amount of subcarriers to be shared in full-reuse. Constraint

[C2] represents the total bandwidth limitation ($B_{tot} = N_s B$). Constraint [C3] corresponds to the BER-outage probability where $\Pr\{X \mid Y\}$ stands for the probability of X knowing Y. The BER $\beta_{b_u,u,s}$ is defined in (6). Finally, [C4] is relative to the rate-outage probability and r_u is given in (7).

IV. Proposed Resource Allocation Scheme

The optimization problem (9) is not tractable because of the large number of variables that need to be jointly optimized in addition to the inherent discrete nature of variables $[b_u]$ and $[f_u]$. Here we propose to transform this problem into an easier one for which a sub-optimal solution can be obtained numerically with a reasonable complexity.

First, we carry out the BS assignment $[b_u]$ separately from the remaining resource allocation. Then we use the different constraints in (9) in order to write $[M_u]$ and $[B_u]$ versus $[b_u]$ and $[f_u]$. Finally, we suggest a threshold-based method for choosing the reuse-scheme $[f_u]$ and we provide a new formulation of the previous optimization problem.

A. Base-Station Assignment

We propose to assign each user to the BS that exhibits the highest shadowed path-loss with that user as follows

$$b_u = \begin{cases} 1 & \text{if} \quad \overline{g}_{1,u} > \overline{g}_{2,u}, \\ 2 & \text{if} \quad \overline{g}_{1,u} \leq \overline{g}_{2,u}. \end{cases} \tag{10}$$

This implies that some users are subject to handover depending on their distances from BSs and on their shadowing realizations. Since the shadowed path-loss varies slowly, handover is not frequent.

B. Rate Adaptation

After the above-described BS assignment, we derive the expression of the modulation order M_u for each user starting from the BER-outage condition [C3] in (9). Replacing (6) into [C3] gives

$$\Pr\left\{0.2\exp\left(\frac{-1.6\,\gamma_{b_u,u,s}}{M_u-1}\right) > \beta\right\} = \mathcal{P}_\beta. \tag{11}$$

Here $\Pr\{\,.\mid(\overline{g}_{1,u},\overline{g}_{2,u})\}$ is replaced by $\Pr\{\,.\,\}$ for notation compactness. It follows from (5) and (11) that

$$\Pr\left\{\frac{\gamma_0\; g_{b_u,u,s}}{1+(2-f_u)\gamma_0\; g_{\overline{b}_u,u,s}} < \frac{M_u-1}{\Gamma_0}\right\} = \mathcal{P}_\beta \tag{12}$$

with

$$\Gamma_0 = -\log(5\beta)/1.6. \tag{13}$$

Depending on the reuse-factor f_u of the considered user, we have one of the following cases:

- **Case 1**– User in partial-reuse:

In this case we have $f_u = 2$ and we prove[1] that

$$M_u = 1 + \frac{\gamma_0\;\overline{g}_{b_u,u}}{F\;\Gamma_0} \tag{14}$$

[1]Proof details are not provided here due to limited room.

with

$$F = -1/\log(1-\mathcal{P}_\beta). \tag{15}$$

The constant F represents the traditional *power fading margin* that guarantees the target outage probability $\mathcal{P}_\beta$ under Rayleigh fading. The parameter Γ_0 is usually called the *coding gap* [18]. This simple result in (14) is due to the absence of interference when user u is assigned to partially-reused subcarriers.

- **Case 2**– User in full-reuse:

In this case we have $f_u = 1$ and we prove[1] that M_u is the solution of the following non-linear equation

$$\frac{\Gamma_0(M_u-1)}{\gamma_0\;\overline{g}_{b_u,u}} = \frac{1}{F} - \log\left(1+\frac{\overline{g}_{\overline{b}_u,u}\;\Gamma_0\;(M_u-1)}{\overline{g}_{b_u,u}}\right). \tag{16}$$

Notice that by setting the interference term $\overline{g}_{\overline{b}_u,u}$ to zero in the previous equation we retrieve (14). Numerical resolution of (16) taking into account realistic values for the involved parameters shows that $(M_u-1)\Gamma_0\frac{\overline{g}_{\overline{b}_u,u}}{\overline{g}_{b_u,u}} \ll 1$. This allows us to use the approximation $\log(1+x) \simeq x$ for $x \ll 1$ in (16) to obtain the following expression

$$M_u \simeq 1 + \frac{\gamma_0\;\overline{g}_{b_u,u}}{(1+\gamma_0\;\overline{g}_{\overline{b}_u,u})F\;\Gamma_0}. \tag{17}$$

This important result can be interpreted in two different manners. In presence of interference ($\overline{g}_{\overline{b}_u,u} \neq 0$), the fading margin F must be increased by a factor $(1+\gamma_0\;\overline{g}_{\overline{b}_u,u})$. By comparing (17) to (14) we can also consider that the average SNR $\gamma_0\;\overline{g}_{b_u,u}$ is replaced by a kind of average SINR $\gamma_0\;\overline{g}_{b_u,u}/(1+\gamma_0\;\overline{g}_{\overline{b}_u,u})$. In this case, the useful average term $\gamma_0\;\overline{g}_{b_u,u}$ in (17) is reduced by a factor F while the interfering average term $\gamma_0\;\overline{g}_{\overline{b}_u,u}$ is kept unchanged (worst case).

In both cases above, i.e. the partial-reuse and the full-reuse, using $(2-f_u)$ as an indicator function for the reuse-scheme of user u leads to the following equation

$$M_u = 1 + \frac{\gamma_0\;\overline{g}_{b_u,u}}{\left(1+(2-f_u)\gamma_0\;\overline{g}_{\overline{b}_u,u}\right)F\;\Gamma_0}. \tag{18}$$

This equation provides a relationship between M_u and f_u given the BS assignment b_u defined in (10).

C. Rate-Outage Characterization

It is obvious from (18) that the allocated modulation order depends on the shadowing realization (CSI). When a low M_u is obtained, the rate r_u of the concerned user can be enforced according to (7) by increasing the allocated bandwidth B_u. However, we expect that such user will penalize the overall system performance due excessive spectral requirement. To avoid this situation, we suggest the introduction of a *cut-off threshold* M_c so that no bandwidth is allocated to user u if $M_u < M_c$. In this case, this user falls in rate-outage. The threshold M_c allows us to adjust the rate-outage probability so that the constraint [C4] is met. By introducing M_c, the constraint [C4] becomes

$$\Pr\{M_u < M_c\} \leq \mathcal{P}_r, \;\; \forall u. \tag{19}$$

Using (18) we prove that the left-hand side of (19), which is the rate-outage probability of user u denoted by $\mathcal{P}_r(u)$, is

$$\mathcal{P}_r(u) = \left\{ \begin{array}{ll} \frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left(\frac{10}{\sqrt{2}\sigma}\log_{10}\frac{(M_c-1)x_{b_u,u}^{\alpha}}{AG_0\,\gamma_0}\right), & f_u=2, \\ \frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left(\frac{10}{2\sigma}\log_{10}\frac{(M_c-1)x_{b_u,u}^{\alpha}}{A\,x_{\overline{b}_u,u}^{\alpha}}\right), & f_u=1. \end{array}\right. \tag{20}$$

with $A = 1/(F\Gamma_0)$. The dependency of $\mathcal{P}_r(u)$ on the reuse-factor f_u results from the fact that M_u also depends on f_u according to (18). Note that this probability increases with the threshold M_c and depends on user location through distances to BSs $x_{b_u,u}$ and $x_{\overline{b}_u,u}$. When $f_u = 2$ (partial-reuse users), $\mathcal{P}_r(u)$ increases with $x_{b_u,u}$. So, to satisfy (19) for all users with $f_u = 2$, the maximum value of M_c must be chosen considering the worst-case user, i.e. an edge user with $x_{b_u,u} = R$ (cell radius). This means that M_c is the solution of

$$\frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left(\frac{10}{\sqrt{2}\sigma}\log_{10}\frac{(M_c-1)R^{\,\alpha}}{AG_0\,\gamma_0}\right) = \mathcal{P}_r. \tag{21}$$

For full-reuse users ($f_u = 1$), the rate-outage probability (21) increases with the distance ratio $x_{b_u,u}/x_{\overline{b}_u,u}$. Here the worst-case user corresponds to user u with $x_{b_u,u} = R, x_{\overline{b}_u,u} = (R + d)$ where d is the inter-BS distance. So, for users with $f_u = 1$ the threshold M_c is the solution of

$$\frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left(\frac{10}{2\sigma}\log_{10}\frac{(M_c-1)R^{\,\alpha}}{A\,(R+d)^{\,\alpha}}\right) = \mathcal{P}_r. \tag{22}$$

From (21) and (22) it follows

$$M_c = \left\{ \begin{array}{ll} 1+\frac{G_0\gamma_0}{R^{\alpha}\Gamma_0 F}10^{0.1(\sqrt{2}\,\sigma)\,\mathrm{erf}^{-1}(2\mathcal{P}_r-1)} & f_u=2, \\ 1+\frac{(R+d)^{\alpha}}{R^{\alpha}\Gamma_0 F}10^{0.1(2\,\sigma)\,\mathrm{erf}^{-1}(2\mathcal{P}_r-1)} & f_u=1. \end{array}\right. \tag{23}$$

To understand this result consider the case $f_u = 2$ and notice that $G_0\gamma_0/(R^{\alpha}\Gamma_0 F)$ is the effective SNR on cell edge when the shadowing is turned-off ($\sigma = 0$). The quantity $\log_2(1 + G_0\gamma_0/(R^{\alpha}\Gamma_0 F))$ represents the allocated data rate for an edge user without shadowing. So, the term $10^{0.1(\sqrt{2}\,\sigma)\,\mathrm{erf}^{-1}(2\mathcal{P}_r-1)}$ in (23) reduces the effective SNR ($10^{0.1(\sqrt{2}\,\sigma)\,\mathrm{erf}^{-1}(2\mathcal{P}_r-1)} < 1$) to take into account the shadowing effect and guarantee a rate-outage probability bounded by $\mathcal{P}_r$. We conclude that $10^{-0.1(\sqrt{2}\,\sigma)\,\mathrm{erf}^{-1}(2\mathcal{P}_r-1)}$ can be considered as a *shadowing margin* related to rate-outage compared to the fading margin F which is related to BER-outage.

By consequent, user u achieves the following rate

$$\begin{array}{lll} M_u \geq M_c \;\Rightarrow & r_u = B_u \log_2 M_u \text{ with proba.} \geq 1-\mathcal{P}_r, \\ M_u < M_c \;\Rightarrow & r_u = 0 \;(B_u = 0) \text{ with proba.} \leq \mathcal{P}_r. \end{array} \tag{24}$$

The last equation is a relationship between the cut-off threshold M_c and the reuse-factor f_u. So, depending on their CSI (shadowed path-loss), users having a poor expected modulation order are forced to be in rate-outage by allocating no bandwidth to them. Now we derive the optimal bandwidth allocation for out-of-outage users taking into account the common rate constraint.

D. Bandwidth Allocation

Users who are not in rate-outage get a common rate r_c that we want to maximize, i.e. $M_u > M_c \;\Rightarrow\; B_u \log_2 M_u = r_c$ for all u. Thus, from (18) and (24) we get

$$B_u = \left\{ \begin{array}{ll} 0 & M_u < M_c, \\ \frac{r_c}{\log_2\left(1+\frac{\gamma_0\,\overline{g}_{b_u,u}}{\left(1+(2-f_u)\gamma_0\,\overline{g}_{\overline{b}_u,u}\right)F\,\Gamma_0}\right)} & M_u \geq M_c. \end{array}\right. \tag{25}$$

Let us summarize the results obtained till now concerning the different optimization variables. For each user, BS assignment b_u is decided first according to (10). Then (18) allows us to find the modulation order M_u which depends on the reuse-factor f_u. The rate-outage cut-off threshold M_c can be obtained using (23) and depends in its turn on f_u. Finally, the bandwidth allocation B_u is given by (25) and is f_u-dependent as well. In summary, once the vector $[b_u]$ is set, vectors $[M_u]$ and $[B_u]$ depend only on $[f_u]$. In the following subsection we propose a threshold-based method to assign values to $[f_u]$. Then we show that the considered optimization problem can be re-written versus a unique variable and lends itself to numerical solution.

E. Frequency-Reuse Factor Assignment

We propose the following intuitive two-threshold-based method for frequency-reuse assignment

$$\begin{array}{l} b_u = 1 \Rightarrow f_u = \left\{ \begin{array}{ll} 1 & \text{if} \;\; \overline{g}_{1,u}/\overline{g}_{2,u} \geq t_1, \\ 2 & \text{if} \;\; \overline{g}_{1,u}/\overline{g}_{2,u} < t_1. \end{array}\right. \\ b_u = 2 \Rightarrow f_u = \left\{ \begin{array}{ll} 1 & \text{if} \;\; \overline{g}_{2,u}/\overline{g}_{1,u} \geq t_2, \\ 2 & \text{if} \;\; \overline{g}_{2,u}/\overline{g}_{1,u} < t_2. \end{array}\right. \end{array} \tag{26}$$

This means that, for a user assigned to BS 1 for example, if the ratio of the useful shadowed path-loss $\overline{g}_{1,u}$ to the interfering one $\overline{g}_{2,u}$ exceeds the threshold t_1, this user is considered as sufficiently isolated from the interfering BS. Consequently, this user is allocated to subcarriers in full-reuse $f_u = 1$. In the opposite case where the ratio $\overline{g}_{1,u}/\overline{g}_{2_u,u}$ is below the threshold t_1, the corresponding user gets partially-reused (interference-free) subcarriers. We call $\overline{g}_{b_u,u}/\overline{g}_{\overline{b}_u,u}$ *the isolation ratio* of user u which according to (10) is always greater than one. Thus, the reuse-thresholds t_1 and t_2 take values in $[1, \infty[$.

Now we can re-formulate the optimization problem (9) using thresholds t_1 and t_2.

F. Optimization Problem Re-formulated

Replacing (25) in (8) and then using (9)-[C2] provides

$$r_c = \frac{B_{tot}}{S_{1,1}+S_{1,2}+S_{2,2}}. \tag{27}$$

with

$$S_{b,f} = \sum_{u:\, M_u > M_c, b_u = b, f_u = f} \frac{1}{\log_2\left(1 + \frac{\gamma_0\, \overline{g}_{b,u}}{\left(1+(2-f)\gamma_0\, \overline{g}_{\overline{b},u}\right) F\, \Gamma_0}\right)}.$$

Using the proposed reuse-scheme (26) the different sums $(S_{b,f})$ defined above can be written versus the reuse thresholds (t_1, t_2) as follows

$$S_{b,1} = \sum_{u:\, \frac{M_u}{M_c}>1, \frac{\overline{g}_{b,u}}{\overline{g}_{\overline{b},u}} \geq t_b} \left[\log_2\left(1 + \frac{\gamma_0\, \overline{g}_{b,u}}{\left(1+\gamma_0\, \overline{g}_{\overline{b},u}\right) F\, \Gamma_0}\right)\right]^{-1}.$$

$$S_{b,2} = \sum_{u:\, \frac{M_u}{M_c}>1, \frac{\overline{g}_{b,u}}{\overline{g}_{\overline{b},u}} < t_b} \left[\log_2\left(1 + \frac{\gamma_0\, \overline{g}_{b,u}}{F\, \Gamma_0}\right)\right]^{-1}.$$

Thus, maximizing r_c is equivalent to minimizing $(S_{1,1}+S_{1,2}+S_{2,2})$ with respect to thresholds (t_1, t_2). Constraint [C1] in (9) can be replaced by $S_{1,1} = S_{2,1}$. All the remaining constraints [C2], [C3] and [C4] are already taken into account in deriving equations (27), (18) and (23) respectively. Therefore, the optimization problem (9) becomes simply

$$\min_{(t_1,t_2)} \; (S_{1,1} + S_{1,2} + S_{2,2}) \quad \text{subject to} \;\; S_{1,1} = S_{2,1}. \tag{28}$$

Now we show that (28) can be further simplified to an unconstrained optimization problem. Remember that the constraint $S_{1,1} = S_{2,1}$ was derived from (9)-[C1] stating that the two BSs must agree on the amount of bandwidth to be shared with reuse factor one. For a given value of t_1, equation $S_{1,1} = S_{2,1}$ provides a way to deduce the corresponding value of t_2. This relationship between t_1 and t_2 can be symbolized by a function $s(.)$ defined as follows

$$s : t_1 \longmapsto t_2 : \; S_{1,1}(t_1) = S_{2,1}(t_2) \tag{29}$$

where the notation $S_{b,1}(t_b)$ emphasizes on the fact that $S_{b,1}$ depends on t_b. This allows us to transform (28) into the following unconstrained problem

$$\min_{t_1} \; \left[S_{1,1}(t_1) + S_{1,2}(t_1) + S_{2,2}(s(t_1))\right]. \tag{30}$$

This problem can be solved numerically by linear search on threshold t_1 as shown later in section VI.

V. Achieved Performance

Let r_c^* be the maximum common rate corresponding to the optimal reuse threshold t_1^* found by resolving (30). From (27) we can write

$$r_c^* = \frac{B_{tot}}{S_{1,1}(t_1^*) + S_{1,2}(t_1^*) + S_{2,2}(s(t_1^*))}. \tag{31}$$

Then, the system-wide spectral efficiency is given by

$$\eta = \frac{\sum_u r_u}{B_{tot}} = \frac{N_p r_c^*}{B_{tot}} \tag{32}$$

where N_p represents the number of provisioned users (users who are not in rate-outage)

$$N_p = \sum_{u:\, \frac{M_u}{M_c}>1} 1. \tag{33}$$

TABLE I
Simulation parameters' values.

Center frequency f_c	3.5	GHz
Total bandwidth B_{tot}	20	MHz
Noise spectral density N_0	-174	dBm/Hz
Path-loss exponent α	3.6	-
Shadowing standard deviation σ	7	dB
Target BER β	10^{-2}	-
Maximum BER-outage probability $\mathcal{P}_\beta$	5	%
Maximum rate-probability $\mathcal{P}_r$	1	%
Cell radius R	150	m
Inter-BS distance d	250	m

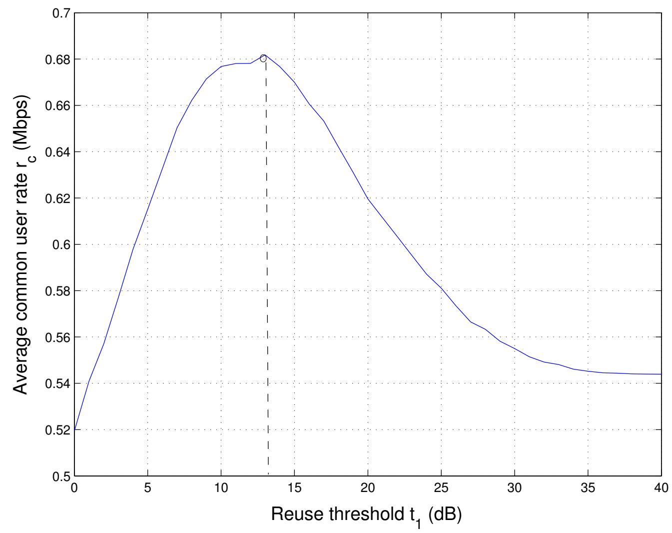

Fig. 1. Average common user rate versus reuse threshold t_1 in dB for $P_{tot} = 0.5$ W and $N_u = 12$ users.

Note that N_p depends implicitly on the reuse-threshold t_1 since M_u depends on f_u according to (18). This observation shows the difference between maximizing the system-wide spectral efficiency and maximizing a common user rate. In the following section the average achieved performance is evaluated by simulation.

VI. Numerical Results

Consider the parameter setting of Table I. Users are randomly but uniformly distributed over the two-cell area.

First we plot in Figure 1 the average value $\overline{r_c}$ of the common rate (27) versus the reuse-threshold t_1 for a fixed total power $P_{tot} = 0.5$ W per BS. The second threshold t_2 is obtained from (29). For $t_1 = 0$ dB, all subcarriers in both cells are in full-reuse. This gives the worst user-rate because all users are subject to ICI. When t_1 increases, the most "isolated" users, i.e. users with the highest isolation ratio $\overline{g}_{b_u,u}/\overline{g}_{\overline{b}_u,u}$, are moved from full-reuse to partial-reuse scheme. Without the effect of random shadowing, these correspond to border users for whom the interfering power is comparable to the useful one. But, the presence of random shadowing makes it difficult to predict the locations of users who are switched to

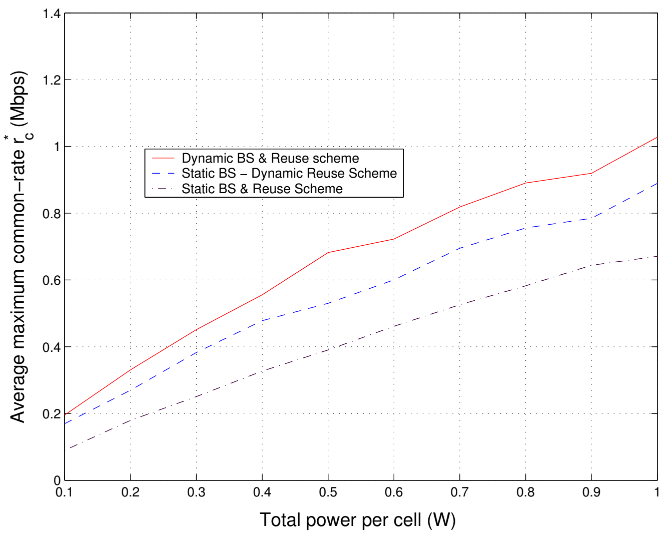

Fig. 2. Average maximum common rate versus total power per BS for $N_u = 12$ users.

partial-reuse as t_1 increases. Beyond a given value of t_1, all users become in partial-reuse and each one of the available subcarriers is exclusively used in one of the two cells. For a particular value of t_1 (about 13 dB in the example of Figure 1), the average common rate attains a global maximum which corresponds to the expectation of r_c^*.

In Figure 2, the total power P_{tot} is varied and the obtained average maximum common rate $\overline{r_c^*}$ is plotted. In this figure, the upper curve (solid-line) corresponds to $\overline{r_c^*}$ obtained with the proposed BS, bandwidth and rate allocation method. To evaluate the benefit of dynamic BS assignment (10), the later is deactivated in the case of the dashed-line curve. Static BS assignment means that each user is assigned to the nearest BS and that no handover does take place. Thus, some degradation in $\overline{r_c^*}$, with respect to the dynamic BS and reuse scheme (solid-line curve), can be observed especially for large values of P_{tot}. Finally, if both BS assignment and frequency-reuse scheme are made static, i.e. no handover and all subcarriers are partially-reused ($f_u = 2$ for all u), a substantial degradation in $\overline{r_c^*}$ takes place.

VII. Conclusion

In this paper, the problem of fair QoS provision in a two-cell OFDMA downlink system was considered. The QoS constraint was defined by a target BER associated with a maximum BER-outage probability. The goal was to maximize a common data rate offered to each user subject to a bounded rate-outage probability. A partial CSI, the shadowed path-loss, was assumed to reduce the feedback overhead and the complexity of the allocation algorithm. The frequency-reuse partitioning was adopted as an inter-cell interference mitigation technique. A simple threshold-based base-station and frequency-reuse-factor assignment method was proposed. This allowed us to transform the considered optimization problem into an unconstrained minimization problem that we resolved numerically. Numerical results allowed us to validate the threshold-based reuse-scheme assignment. They also showed the benefit, in terms of average user rate, of the dynamic BS assignment (handover) and the dynamic frequency-reuse partitioning.

In our analysis, continuous modulation and bandwidth allocation were assumed. Thus, future work will focus on studying the effect of subcarrier-based allocation as well as discrete MQAM modulation. We will consider also a distributed version of the proposed allocation method that yields further reduction in inter-BS signaling.

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