FAIR SERVICE PROVISION IN OFDMA WITH PARTIAL CHANNEL-STATE INFORMATION

Ayman Alsawah, Inbar Fijalkow

ETIS, CNRS, ENSEA, Univ Cergy-Pontoise, F-95000 Cergy-Pontoise
Email: {ayman.alsawah, inbar.fijalkow}@ensea.fr

ABSTRACT

We address the problem of resource allocation on an OFDMA downlink under fairness constraints with limited Channel State Information (CSI). Target QoS corresponds to a minimum user data rate, a target bit-error rate and a maximum outage probability. The only available CSI is the channel average gain of each user. This partial CSI can be viewed as a *shadowed pathloss* that yields a modified user distribution on which resource allocation is based. Under these constraints, we provide the optimal resource allocation that maximizes the user rate. Compared to full-CSI-based allocation schemes, our solution offers a significant complexity and feedback reduction as well as a good robustness to CSI estimation errors.

1. INTRODUCTION

Resource allocation in OFDMA under fairness constraints is an active research area [1–4]. Depending on the application, the proposed solutions differ in the fairness constraints and in the objective function. However, the common point is that some available *Channel State Information* (CSI) is used in order to maximize the objective function. The optimization operates on some degrees of freedom like the subcarrier, rate and power allocation schemes. Authors often consider the case of full CSI with continuous modulation and zero-outage [1–4]. Some authors [1, 2] consider the error-free Shannon capacity. Such information-theoretical approaches provide upper-bounds on the achievable system performance but are not directly connected to real implementations. Other authors [3, 4] consider a non-zero BER with M-QAM constellations. In this case, an M-QAM BER approximation, such as the SNR coding Gap [5], is used. Another example with encoded M-QAM can be found in [6]. Obviously, the ultimate optimal performance is obtained if we jointly optimize the available degrees of freedom under full CSI knowledge. This allows us to exploit the different forms of diversity like time, frequency or multiuser diversity. Unfortunately, finding the optimal solution involves excessive computational complexity [1]. Hence, several suboptimal solutions have been proposed. Some are based on separating the subcarrier allocation step from the power allocation step as in [3]. Others [1] assume equal powers to simplify the subcarrier assignment. Nevertheless, full CSI assumption remains unrealistic in practical systems because of the excessive feedback bandwidth it requires, especially for a large number of users and subcarriers. Several methods [7] were proposed to reduce the amount of required feedback. Moreover, whatever the CSI estimation method is, the obtained CSI is imperfect due to estimation errors, feedback errors, feedback delay and quantization noise. Thus, the goal of the present work is to propose a low-complexity resource allocation algorithm which is robust to CSI accuracy assuming a simple partial CSI.

This paper focuses on the downlink of a single-cell OFDMA system with adaptive modulation and subcarrier allocation. Fairness constraints correspond to a target Quality of Service (QoS) common to all the users and defined by a minimum data rate and a target Bit-Error Rate (BER). The considered channel model includes pathloss, log-normal shadowing and Rayleigh fading. We suppose that the only CSI available to the Base Station (BS) is the average channel power gains for the different users. This CSI can be estimated by averaging the received power over the subcarriers assuming equal subcarriers' powers. It can be viewed as a *shadowed pathloss* that corresponds to a modified user distribution. This partial CSI yields a feedback overhead reduction factor equal to the number of subcarriers compared to the overhead required for full-CSI. Resource allocation is carried out based on the notion of *shadowed distance* that we introduce. Thus, we provide the optimal resource allocation that maximizes user data rate. Simulations allows us to compare our method to other existing ones in terms of average spectral efficiency. They also show that the overall outage performance of the proposed method is robust to CSI estimation errors.

2. SYSTEM MODEL

Consider an OFDMA downlink from a BS to U uniformly-distributed users in a single circular cell of radius R. A total transmit power P_{tot} is equally partitioned over S subcarriers. The transmitted signal received by user u experiences a frequency-selective slow-fading channel characterized by the power gains $g_{u,s}$ ($s = 1, .., S$) over the set of subcarriers. Each $g_{u,s}$ accounts for a deterministic pathloss $G(x_u)$, that

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depends on the distance x_u of user u to the BS, in addition to a *log-normal shadowing* $10^{\,0.1\,\xi_u}$ and a multipath *Rayleigh fading* $\phi^2_{u,s}$. So, the received *Signal-to-Noise Ratio* (SNR) at the u^{th} user on subcarrier s is

$$\gamma_{u,s} = \frac{P_{tot}\ g_{u,s}}{SBN_0} = \frac{P_{tot}}{SBN_0}\ G(x_u)10^{\,0.1\,\xi_u}\ \phi^2_{u,s} \tag{1}$$

where B is the subcarrier spacing and N_0 is the AWGN power spectral density. The pathloss $G(x_u)$ represents the long-term average, called also the *area mean*, of the channel power gain at distance x_u. Thus, the quantity

$$\bar{\bar{\gamma}}_u = \frac{P_{tot}}{SBN_0}\ G(x_u) \tag{2}$$

is the *area-mean received SNR* at user u. We assume that the pathloss follows the *exponent model* [8] defined by $G(x_u) = G_0/x_u^\alpha$ with $G_0 = (c/4\pi f_c)^2$ (f_c is the center frequency and c is the light speed). The log-normal shadowing $10^{\,0.1\,\xi_u}$ is the *local-mean* gain characterized by σ, the standard deviation of the zero-mean Gaussian random variable ξ_u. The *local-mean received SNR*, or the shadowed SNR, can be defined by

$$\bar{\gamma}_u = \frac{P_{tot}}{SBN_0}\ G(x_u)\ 10^{\,0.1\,\xi_u}. \tag{3}$$

It follows the log-normal distribution $\mathcal{LN}(10\log_{10}\bar{\bar{\gamma}}_u, \sigma^2)$. We define the *Shadowed Pathloss* for user u by

$$\mathcal{G}_u = G(x_u)\ 10^{\,0.1\,\xi_u} = (G_0/x_u^\alpha)\ 10^{\,0.1\,\xi_u}. \tag{4}$$

This quantity can be estimated in practice by averaging the received power over the subcarriers during one or more dedicated OFDM training symbols. The reason for calling $\mathcal{G}_u$ the shadowed pathloss is that a *shadowed user-distance* d_u can be introduced so that $\mathcal{G}_u$ is also obtained by the same pathloss exponent model as follows

$$\mathcal{G}_u = G_0/d_u^\alpha. \tag{5}$$

Thus, from (4) and (5) we get

$$d_u = x_u\ 10^{\,-0.1\,\xi_u/\alpha}. \tag{6}$$

Replacing user actual distance by the shadowed one allows us to make abstraction of the shadowing. In the following, we consider that **the available CSI about user u is the shadowed distance d_u defined in (6)**. Based on this CSI, the BS allocates system resources to maximize user rate while satisfying the required QoS. The required QoS is defined by the minimum data rate per user D_0, the target BER b and the maximum BER-outage probability ε. The reason why the shadowed distance d_u is considered rather than the shadowed pathloss (4) is that the proposed algorithm is based on user distribution. The resource allocation is done on a frame-basis with a frame length of L_f OFDM symbols. Finally, we assume that the set $\mathcal{M} = \{M_1, ..., M_Q\}$ of available M-QAM orders is an ordered set ($M_q > M_{q+1}$).

3. OPTIMIZATION PROBLEM FORMULATION

Since the shadowed distance d_u is not a frequency-selective CSI, the subcarrier allocation is transformed into a bandwidth allocation. So, resource allocation consists in deciding how many subcarriers each user does need and which M-QAM constellation has to be used on them. We introduce the vector $W = [W(1), ..., W(U)]$ where $W(u)$ represents the number of subcarriers allocated to user u. We assume that the same constellation $M(u)$ is used on the $W(u)$ subcarriers. The optimal $M(u)$ depends on the target BER b, on the maximum outage probability ε and on the available CSI d_u. With uncoded M-QAM, user u achieves a total data rate

$$r_u = W(u)B\log_2 M(u). \tag{7}$$

Thus, our optimization problem can be formulated as follows

$$\begin{aligned} &\max_W D \\ &\text{subject to} \quad \text{[c1]: } W(u) \in]0, S[\ \ \forall u, \\ &\qquad\qquad\quad \text{[c2]: } \textstyle\sum_{u=1}^U W(u) \le S, \\ &\qquad\qquad\quad \text{[c3]: } r_u = D\ \ \forall u, \\ &\qquad\qquad\quad \text{[c4]: } r_u \ge D_0\ \ \forall u. \end{aligned} \tag{8}$$

From [c3], the rate D is the data rate common to all users. The condition [c4] represents the minimum data rate constraint. In the following section we derive analytic expressions for the maximum achievable rate D_{max} and the corresponding optimal subcarrier and rate allocation.

4. OPTIMAL RESOURCE ALLOCATION

Maximizing the common rate corresponds to the choice of the highest possible constellation-order for each user. But, the choice of constellation is constrained by the maximum outage probability. The BER-outage probability of user u is $p_{out}(u) = \text{Proba}\left[\beta_{M(u)}(\gamma_{u,s}) > b\right]$ where $\beta_{M(u)}(\gamma)$ is the achieved BER versus the SNR γ and the modulation order $M(u)$. Since $\beta_{M(u)}(\gamma)$ is decreasing with respect to γ, we have $p_{out}(u) = \text{Proba}\left[\ \gamma_{u,s} < \beta^{-1}_{M(u)}(b)\right]$ with $\beta^{-1}_{M(u)}(.)$ being the inverse function of $\beta_{M(u)}(.)$ that provides the SNR threshold for BER=b. This probability can be expressed using the *cumulative distribution function* $F_{\gamma_{u,s}}(.)$ of $\gamma_{u,s}$. Conditionally to the shadowed SNR (3), $\gamma_{u,s}$ follows a Chi-square law so that $F_{\gamma_{u,s}}(\gamma) = 1 - \exp(\gamma/\bar{\gamma}_u)$. Thus we get

$$p_{out}(u) = 1 - \exp\left(-\beta^{-1}_{M(u)}(b)/\bar{\gamma}_{u,s}\right). \tag{9}$$

Using equations (5) and (4), the shadowed SNR (3) becomes

$$\bar{\gamma}_u = P_{tot}\ G_0/(SBN_0 d_u^\alpha). \tag{10}$$

Replacing (10) into (9) provides

$$p_{out}(u) = 1 - \exp\left(-\beta^{-1}_{M(u)}(b)\ SBN_0 d_u^\alpha/(P_{tot}G_0)\right). \tag{11}$$

For a given $M(u)$, this probability increases with d_u. Therefore, the maximum outage probability constraint $p_{out}(u) \leq \varepsilon$ means that each constellation M_q-QAM can be used up to a maximum distance R_q which is the solution of the equation

$$1 - \exp\left(-\beta_{M_q}^{-1}(b)\ SBN_0\ R_q^{\alpha}/(P_{tot}\ G_0)\right) = \varepsilon.$$

We obtain

$$R_q = \left[\frac{(P_{tot}/F)\ G_0}{SBN_0\ \beta_{M_q}^{-1}(b)}\right]^{1/\alpha} \tag{12}$$

with the parameter F given by

$$F = -1/\log(1-\varepsilon). \tag{13}$$

This F represents the *fading power margin*. In (12), the SNR-threshold function $\beta_{M_q}^{-1}(b)$ is increasing with M_q (a higher-order modulation requires higher SNR for the same BER). So, for the complete set of available constellations we have $R_1 < R_2 < ... < R_Q$. Remember that maximizing the common rate requires using for each user the constellation of the highest possible order. Consequently, the M_q-QAM constellation must be allocated to users whose shadowed distances d_u are in $]R_{q-1}, R_q]$ with $R_0 = 0$. This defines the optimal rate allocation as follows

$$M(u) = \max_q \left\{M_q \in \mathcal{M} :\ d_u \leq R_q\right\}. \tag{14}$$

Thus, each constellation M_q-QAM covers an *annular zone* of internal (resp. external) radius R_{q-1} (resp. R_q). In the following, the q^{th} annular zone is referred to as *zone* q.

The aim now is to find the optimal subcarrier allocation. We saw that the subcarrier allocation under our assumptions is reduced to a user-wise bandwidth allocation defined by $W = [W(1), ..., W(U)]$. Suppose that users achieve an *unknown* rate D_{max} each ($r_u = D_{max},\ \forall u$). From (7) we get

$$W(u) = D_{max}/(B\ \log_2 M(u)). \tag{15}$$

From (14), the optimal constellation $M(u)$ is equal to M_q for users of zone q. Consequently, each user in zone q needs $D_{max}/(B \log_2 M_q)$ subcarriers. The ratio of the number of users contained in zone q to the total number of users is

$$u_q = \frac{1}{U}\left|\left\{u = 1, .., U : R_{q-1} < x_u 10^{-\frac{0.1\xi_u}{\alpha}} \leq R_q\right\}\right| \tag{16}$$

where $|\mathcal{A}|$ is the cardinal of set $\mathcal{A}$. Thus, the number of subcarriers required by zone q is

$$S_q = D_{max} U\ u_q/(B \log_2 M_q). \tag{17}$$

So, from the constraint of the total number of subcarriers $\sum_{q=1}^{Q} S_q = S$ and from (17) it follows

$$D_{max} = \frac{BS}{U \sum_{q=1}^{Q} u_q/\log_2 M_q} \tag{18}$$

which is the maximum achievable common rate. By substituting (18) back into (17) we finally get

$$S_q = \frac{u_q/\log_2 M_q}{\sum_{k=1}^{Q} u_k/\log_2 M_k}\ S. \tag{19}$$

Note that S_q is a random variable because u_q is. This means that the bandwidth reservation depends on the shadowing realization. So, the BS updates the variables S_q's each time it gets a new feedback about users' shadowed-distances.

In the following section we evaluate the achievable performance in terms of rate-outage and spectral efficiency.

5. ACHIEVABLE PERFORMANCE

Although users are actually located inside the cell of radius R, their shadowed distances may be greater than the range of the lowest-order modulation R_Q that can be obtained from (12). As long as the user shadowed-distance is smaller than R_Q, this user can be served with the required QoS. In the opposite case, this user is in *rate-outage*. We define the *rate-outage probability* for user u by $\rho_u = \text{Proba}\,[d_u > R_Q]$. This probability can be derived given the statistics of d_u defined in (6). It is easier to consider $10\log_{10} d_u = 10\log_{10} x_u - \xi_u/\alpha$ which follows a Gaussian law $\mathcal{N}(10\log_{10} x_u, \sigma^2/\alpha^2)$. The corresponding cumulative distribution function is given by $F_{10\log_{10} d_u}(y) = 0.5 + 0.5\ \text{erf}\left((\alpha/\sigma\sqrt{2})(y - 10\log_{10} x_u)\right)$ where erf(.) is the *error function*. So, we can write

$$\begin{aligned}\rho_u &= 1 - F_{10\log_{10} d_u}(10\log_{10} R_Q)\\ &= 0.5 - 0.5\ \text{erf}\left((10\ \alpha/\sigma\sqrt{2})\log_{10}(R_Q/x_u)\right).\end{aligned}$$

This probability reaches its maximum for edge users $x_u = R$

$$\rho_{max} = 0.5 - 0.5\ \text{erf}\Big((10\ \alpha/\sigma\sqrt{2})\log_{10}(R_Q/R)\Big). \tag{20}$$

Note that the number of served users is $U_{srv} = U\sum_{q=1}^{Q} u_q$. These users achieve a total data rate of $U_{srv} D_{max}$ using the total bandwidth BS. So, from (18), the spectral efficiency is

$$\eta = \frac{\sum_{q=1}^{Q} u_q}{\sum_{q=1}^{Q} u_q/\log_2 M_q}. \tag{21}$$

The constraint of the minimum data rate per user D_0 has not been considered yet. Having $D_{max} \geq D_0$ along with (18) gives an *upper-bound on the bearable number of users*

$$U_{max} = \frac{BS}{D_0\ \sum_{q=1}^{Q} u_q/\log_2 M_q}. \tag{22}$$

This U_{max} corresponds to *the maximum system load*. When the system is fully-loaded ($U = U_{max}$), it cannot offer to each user more than the minimum rate D_0 (with the outage probability calculated above) and any additional user that requests an access to the service is rejected.

The next section describes the proposed resource allocation algorithm from a practical point of view.

6. PRACTICAL ALLOCATION ALGORITHM

In practice, only an integer number of subcarriers can be assigned to a given modulation zone. Thus, the S_q's defined in (19) have to be rounded to integer numbers

$$\tilde{S}_q = \mathcal{I}(S_q) = \mathcal{I}\left(\frac{u_q/\log_2 M_q}{\sum_{k=1}^{Q} u_k/\log_2 M_k}\, S\right) \qquad (23)$$

where $\mathcal{I}(x)$ is the nearest integer to x. These $\tilde{S}_q$'s define Q zones inside the frame where the first zone for example is constituted of slots modulated by the M_1-QAM constellation. From (15), the number of subcarriers that a user u in zone q needs to achieve the common rate is equal to $W(u) = D_{max}/(B\ \log_2 M_q)$. So, another concern is that $W(u)$ is not necessarily an integer. In practice, users are mapped to a frame of length L_f OFDM symbols. With S subcarriers, a frame is composed of SL_f slots. Let N_q be the integer number of slots allocated to each user in zone q. Hence, each user obtains on average N_q/L_f subcarriers per frame. So, to cope with a non-integer $W(u)$, the value of N_q must be chosen so that the difference $|N_q/L_f - W(u)|$ is minimized. This gives

$$N_q = \mathcal{I}(L_f W(u)) = \mathcal{I}(L_f\ D_{max}/(B\ \log_2 M_q)). \qquad (24)$$

Moreover, the actual number of users that can be mapped to the $\tilde{S}_q L_f$ slots of the M_q-QAM zone in the frame is equal to

$$\tilde{U}_q = \tilde{S}_1 \div N_q \qquad (25)$$

where $\div$ is the integer division operator.
Once the zones' radii are obtained from (12), the proposed algorithm, depicted in Fig. 1, consists of the following steps:

• Sort users according to increasing shadowed distance to form a vector $\mathcal{U}$. Then, find U_q, the number of users in each modulation zone.

• Deduce the zone-wise number of subcarriers $\tilde{S}_q$ for $q = 1, ..., Q$ using (23).

• Use (24) and (25) to find the required number of slots per user N_q and the zone-wise number of bearable users $\tilde{U}_q$.

• Map users to the frame as follows: the first $\tilde{U}_1$ users in $\mathcal{U}$ are mapped to the first $\tilde{S}_1$ subcarriers modulated with the highest-order constellation. The first user is granted the first N_1 time slots on the first subcarrier. Then, the next user occupies the next N_1 slots and so on. These operations are repeated for the next $\tilde{U}_2$ users in $\mathcal{U}$, that belong to the second modulation zone, and so on until all slots are occupied.

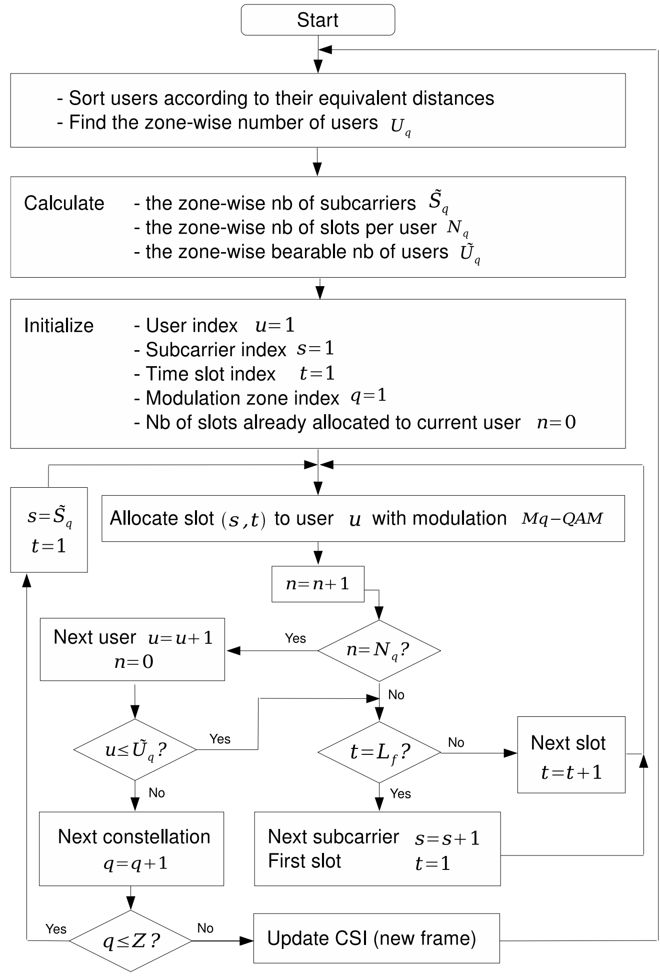

Fig. 1. Flow chart of the resource allocation algorithm.

7. NUMERICAL RESULTS

Table 1 gives the simulation parameters. The modulation set is {64-QAM,16-QAM,QPSK,BPSK}. We define the *Worst-case Average SNR* (WASNR) by $\gamma_{wa} = P_{tot}G_0/(SBN_0\ R^{\alpha})$. From (13), the required fading margin is $F_{dB} \simeq 12.9$ dB. To cover the whole cell a minimum WASNR of $\gamma_{wa} \simeq 19.64$ dB is required. The maximum power $P_{tot} = 10$ W corresponds to $\gamma_{wa} \simeq 25.6$ dB. Hence, the WASNR varies within [20,25] dB. For $U = 10$ users we compare in Fig. 2 the average spectral efficiency of our method to that of two others: the "static" method (no CSI) and the "Max-Min" method (full CSI) [1]. In the "static" method, the required power margin for both shadowing and fading is $F = 14.8$ dB. Without any CSI, the same modulation of order M is used over all subcarriers where M is adapted to the edge user. With $d_u = R$ in (14) we get $M = 2$. Then, the S subcarriers are equally partitioned among the users so that each user achieves a data rate $(S/U)B\log_2 M$. We see that our algorithm offers a spectral efficiency gain compared to the static method. The "Max-Min" method goal is to maximize the minimum user rate. Based on full CSI, it allows an improved spectral efficiency but requires excessive feedback overhead. To make the comparison fair, the CSI is quantized on 3 bits so that the feedback overhead required by the "Max-Min" method becomes $3S$ bits/user against 3 bits/user for our method. As mentioned earlier, the main advantages of our method are its simplicity

Table 1. Simulation parameters' values.

Center frequency f_c	3.5	GHz
Total transmit power P_{tot}	10	W
Number of subcarriers S	256	
Total bandwidth B_{tot}	20	MHz
Frame length L_f	100	symbols
Noise spectral density N_0	-174	dBm/Hz
Pathloss exponent α	3.6	
Shadowing standard deviation σ	5	dB
Target minimum data rate D_0	100	Kbps
Target BER b	10^{-3}	bps
Maximum outage probability ε	0.05	
Target cell radius R	100	m

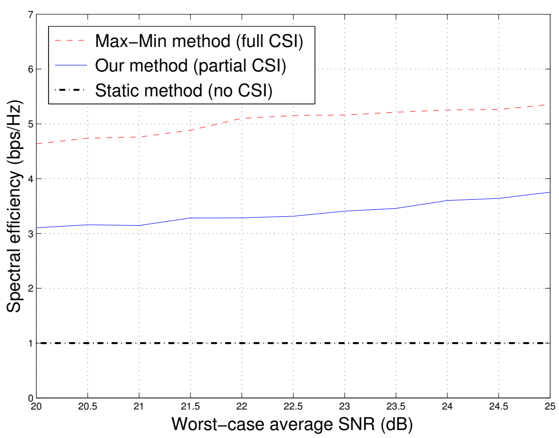

Fig. 2. Average spectral efficiency comparison.

and the limited CSI feedback it requires. The price to be paid is a loss in spectral efficiency compared to the "Max-Min" method as shown in Fig. 2.

To characterize the sensitivity of our algorithm to CSI accuracy, a zero-mean Gaussian error is added to the shadowed distance, that is $\hat{d}_u = d_u + e_u$. The error standard deviation is $\sigma_u = a\ R$ so that a measures the CSI accuracy. Errors on shadowed distances lead to unexpected BER-outage events. We plot in Fig. 3 the *average percentage of users in outage* as an overall performance metric. We notice that the percentage of users in outage does not exceed $4\ \%$ even at $a = 0.5$ which corresponds to a significantly-degraded CSI. This shows the robustness of our algorithm to CSI estimation errors.

8. CONCLUSION

We considered the problem of resource allocation on a single-cell OFDMA downlink under QoS fairness constraints with limited CSI. The available CSI was the users' shadowed distances that we defined. We derived the optimal resource allocation that maximizes the user data rate. Simulations showed that our algorithm yields a spectral efficiency gain compared to traditional static scheme. The loss in average spectral efficiency with respect to a full-CSI-based opportunistic allocation is the price to be paid for the complexity and feedback overhead reduction that our solution offers. Finally, the robustness of our algorithm to CSI estimation errors was shown.

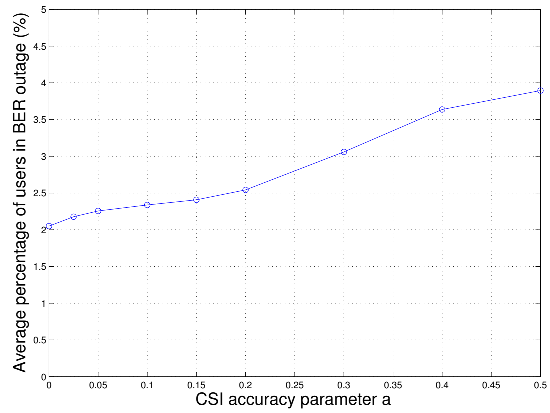

Fig. 3. Effect of CSI accuracy on outage performance.

Future work will focus on deriving analytic expressions for the average shadowed user distribution and extending our approach to the multi-cell case with inter-cell interference.

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