

Rateless Coding for Quasi-Static Fading Channels Using Channel Estimation Accuracy

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Abstract—The design of efficient rateless coding schemes for multicast applications in wireless environments is investigated. First, the rateless paradigm for non-ergodic channels is introduced by making use of the dynamic-decoding nature of rateless codes that allows them to adapt opportunistically the code rate to the channel realization (assumed unknown at the transmitter). The information theoretical limits of such codes can be interpreted in terms of the notion of outage capacity. Then, we consider a quasi-static Rayleigh-fading channel with perfect and imperfect channel state information (CSI) at the receiver. We show that the optimal *consistent measure of information* for decoding with imperfect CSI, is given by the log-likelihood ratio (LLR) of the received bits via a composite (more noisy) channel. The optimization of Raptor codes, which depends on the delay requirements of decoding, is obtained by using *Information content evolution* under Gaussian approximation. Simulation results show that optimized Raptor codes can operate very close to the theoretical limits on a wide range of delay requirements.

I. INTRODUCTION

A very challenging aspect in modern mobile wireless applications is the very strong constraints on the energy consumption. To achieve low energy consumption, it is necessary to make an efficient use of the transmission channels, *i.e.* transmit at the highest possible rate given the channel conditions. One measure that is of particular interest for a communication system is its *throughput*, defined as the average number of data bits accepted at the receiver (or reliably transmitted) per time required for the transmission of a single bit. Reliable transmission is obtained by using error correcting codes that introduce redundancy in order to be robust to transmission errors, and therefore reduces the throughput. Typically, a mobile receiver wants to minimize the transmission time (*i.e.* the battery time), by receiving the information at the highest rate possible for each channel realization.

In multicast scenarios each receiver sees a different channel, leading to different maximal coding rates. Consequently, for fixed rate schemes, where the rate must be chosen according to the worst channel, the receivers with good channels suffer from information delay and rate loss. Thus, there is an unavoidable trade-off between reliability (low coding rate) and efficiency (high coding rate). Alternative strategies have been widely considered. For example, for systems with a feedback channel, the receiver asks for retransmission whenever it fails to recover a message. This approach is known as Automatic Repeat reQuest (ARQ) schemes. Hybrid ARQ (HARQ) is

an enhanced version of ARQ where data is precoded with an error correcting code. This has two opposing effects: (i) the code rate penalty decreases the throughput and (ii) the probability that the transmission succeeds is increased. Incremental Redundancy HARQ (IR-HARQ) adapts the error correcting rate to the channel conditions. The receiver asks the transmitter for additional parity bits when decoding is not successful. Its disadvantage is that it cannot operate below a design rate which is the rate of the mother code.

An alternative to IR-HARQ is the use of *rateless* coding. The transmitter produces a potentially limitless number of independent symbols and the receiver tries to decode the information block as it receives output symbols. When the receiver has collected enough output symbols to recover the message, then it can send an ACK and disconnect, potentially saving transmission time and energy. Whereas, traditional block codes are characterized by their design rate and require puncturing to achieve higher rates, a rateless code achieves this by adapting the number of output symbols. The rateless scheme can be practically implemented with Fountain codes, which are a family of naturally rateless codes, originally introduced for communicating over erasure channels [1]–[3].

In [4], the authors compare an IR-HARQ scheme based on punctured LDPC codes, and a HARQ scheme based on Raptor codes [3] for communication over time-varying channels. They characterized the Maximum-Likelihood (ML) performance of the two schemes by exhibiting for each one, bounds on the asymptotic left tail of the code spectrum. They do not directly address the optimization of Raptor codes, but rather their approach relies on a power allocation strategy. In [5], the use of rateless codes for communication over Rayleigh-fading channel is addressed. Their results show that such codes are very promising. Nevertheless, the authors use codes optimized for a binary input additive white Gaussian noise channel (BIAWGNC) of capacity 1/2 bits, resulting in a wide gap to the channel capacity in the high SNR regime.

In this paper we extend the application of rateless coding to non-ergodic memoryless channels. In particular, we investigate the capability of Raptor codes to select their decoding time dynamically to match the rate of communication to the instantaneous channel capacity. We develop coding strategies for quasi-static (QS) Rayleigh-fading channels, where perfect and imperfect channel estimation is available at the receiver.

First, based on the notion of outage capacity we characterize the optimal trade-off between the coding rate and the decoding time. We propose an optimization method for Raptor codes on quasi-static fading channels. Then we show that the optimal *consistent measure of information* (using channel estimation accuracy) for decoding with imperfect channel estimation is given by the log-likelihood ratio (LLR) of the received bit via a composite (more noisy) channel.

The remainder of this paper is organized as follows. Section II introduces rateless coding for non-ergodic channels and characterizes its ultimate performances. Section III presents Raptor codes and provides an optimization method for fading channels by assuming perfect channel estimation at the receiver. Section IV formalizes the problem with imperfect channel estimation, deriving the optimal LLRs. Simulation results and conclusions are provided in sections V and VI.

II. RATELESS CODING FOR BLOCK FADING CHANNELS

A. Channel model

A difficult aspect of wireless communications is channel fading. Various phenomena such as multipath propagation, terminal mobility and users interference, result in channels with time-varying parameters. Let us review such models for communications over memoryless channels with complex input and output alphabets. A specific instance of the channel is characterized by the transition probability density $W(y|x, H) = \mathcal{CN}(Hx, \sigma_Z^2)$ with channel state $H \in \mathbb{C}$ (cf. [6]). Throughout the paper we assume that the channel state, which is unknown at the transmitter with perfect and imperfect channel state information at the receiver (CSIR), is constant within the transmission block. Furthermore, channel states in different blocks follows flat Rayleigh-fading with $P_H = \mathcal{CN}(0, \sigma_H^2)$. The input symbols X_i belong to a QPSK modulation and the energy per symbol is normalized so that $\mathbb{E}_X\{|X_i|^2\} = P$, where $\mathbb{E}\{\cdot\}$ denotes expectation.

B. The rateless paradigm for non-ergodic channels

Conditioned on the channel realization $H = h$, the instantaneous channel is an AWGNC with capacity

$$C(h) = \max_{P_X} I(X; Y, H = h). \quad (1)$$

Therefore, a quasi-static fading channel can be seen as an AWGN channel where the capacity $C(h)$ is a random variable. In the rateless setting, the inverse of the information rate is proportional to the delay needed to correctly decode the information sent by the encoder. Hence, the number of bits required to decode such code only depends on the current channel draw, *i.e.* decoding in less time when the instantaneous channel realization is good and taking more time to decode when the channel is bad. Thus, a decoder might not support the maximal information delay Δ tolerated and an outage will occur with a certain probability.

We consider outage events induced by information delays (*i.e.* decoding time). Note that with infinite delay, rateless codes guarantees no outage events. This follows from the

fact that if at some point the receiver has not recovered the transmitted message, we do not consider that an outage occurs, but instead that the receiver must collect more data in order to recover the message. In the sequel, we will not consider the coding rate R , rather the delay given by its inverse $\Delta = R^{-1}$.

C. Theoretical limits

We propose to measure the theoretical performances of rateless codes in terms of the probability of decoding delay $P_{\text{wait}}(\Delta)$, defined as the probability that the information sent by the encoder at a given delay Δ (or rate $R = \Delta^{-1}$) be not sufficient for the instantaneous channel realization, and that the receiver must wait for more data.

$$P_{\text{wait}}(\Delta) = \Pr\{I(X; Y, H = h) < \Delta^{-1}\}.$$

Given a system requirement for the delay probability p_{wait} , we define Δ^* as the minimum delay such that for any delay $\Delta \geq \Delta^*$ ($\geq \Delta_{\min}$ delay minimal) the probability $P_{\text{wait}}(\Delta) \leq p_{\text{wait}}$,

$$\Delta^*(p_{\text{wait}}) = \inf\{\Delta \geq \Delta_{\min} : P_{\text{wait}}(\Delta) \leq p_{\text{wait}}\}.$$

The mutual informations with Gaussian and QPSK inputs are

$$I(X_G; Y, H = h) = \log_2(1 + |h|^2 \text{SNR}), \quad (2)$$

$$I(X_{\text{QPSK}}; Y, H = h) = 2J\left(\frac{4|h|^2}{\sigma_Z^2}\right), \quad (3)$$

where $\text{SNR} = \frac{P}{\sigma_Z^2}$ and the function $J(\cdot)$ is defined by:

$$J(m) = 1 - \frac{1}{\sqrt{4\pi m}} \int_{\mathbb{R}} \log_2(1 + e^{-\nu}) \exp\left(-\frac{(\nu - m)^2}{4m}\right) d\nu. \quad (4)$$

The delay probability curves of a Rayleigh-fading channel are represented on Fig. 1. For a $p_{\text{wait}} = 10^{-2}$ and $\text{SNR} = 15\text{dB}$ the curve is approximately at $\Delta^*(p_{\text{wait}}) = 2$. This means that, when the receiver has collected twice the amount of information bits or equivalently when the current coding rate equals $1/2$, the probability that the decoder has to collect more data to be able to decode is less or equal to 10^{-2} .

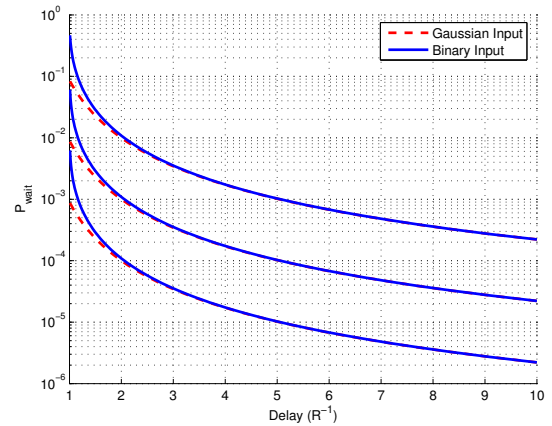


Fig. 1. Theoretical limits of rateless schemes over Rayleigh-fading quasi-static channels for three values of SNR (10dB, 15dB, 20dB)

III. RAPTOR CODES WITH PERFECT CSIR

One practical implementation of rateless codes is Raptor codes. Raptor codes are known *not* to be universal [7] on general binary input memoryless channels other than the BEC, *i.e.* they cannot approach arbitrarily close the capacity *independently* of the channel statistic. However, we will see that they are good candidates to implement rateless coding, since these can operate close to the instantaneous capacity over a wide range of channel gains.

A. Notations

A Raptor code is a concatenation of a high rate block error correcting code called *precode*, and an LT code characterized by its *output degree distribution*. Let K be the number of information symbols, which are precoded to form a codeword of the precode of N *input symbols*. To generate an *output symbol* of the LT code, a degree d is sampled from that distribution. The output symbol of is then formed as the binary sum of an uniformly randomly chosen subset of size d of the input symbols.

Let $\underline{\Omega} = (\Omega_1, \Omega_2, \dots, \Omega_{d_c})$ be the distribution weights on $1, 2, \dots, d_c$ so that each Ω_d denotes the probability of choosing the value d . We denote the output degree distribution using its generator polynomial: $\Omega(x) = \sum_{i=1}^{d_c} \Omega_i x^i$, which is associated with the corresponding edge degree distribution $\omega(x) = \sum_{i=1}^{d_c} \omega_i x^{i-1} = \Omega'(x)/\Omega'(1)$. The input symbols node degree distribution can be approximated by a Poisson distribution with parameter α [3] $I(x) = e^{\alpha(x-1)}$. The associated input edge degree distribution $\iota(x) = \sum_{i=1}^{d_v} \iota_i x^{i-1} = I'(x)/I'(1)$ also equals $e^{\alpha(x-1)}$. In the context of data broadcasting, Raptor codes are of particular interest because all the output symbols being independent, any sufficiently large set of output symbols allows to recover the data with high probability. For more details on the construction see references [2], [3], [8].

B. Optimization problem statement

A Raptor code is entirely characterized by its output degree distribution. The optimization of an output degree distribution arises from the characterization of the decoder. Our concern in this section is to provide an optimal approach for the design of such codes for quasi-static fading channels. We define a cost function for the optimization of the output degree distribution and derive an optimization method that can be stated as a linear problem and thus easily solved by using algorithms such as the simplex algorithm.

1) *Cost function*: We now assume that a system designer wants to guarantee a maximal delay for $(1 - p_{\text{wait}})\%$ of users, which means that any information delay $\Delta \leq \Delta^*(p_{\text{wait}})$ must be tolerated (see Section II-C). Thus, good codes must operate close to the instantaneous capacity of the set of channel realizations $\Lambda(p_{\text{wait}})$, which corresponds to the desired range of information delays. An output degree distribution satisfying this can be found as the solution to the following expression

$$\underline{\Omega}_{\text{opt}}(p_{\text{wait}}) = \arg \inf_{\underline{\Omega} \in \Xi} \sup_{h \in \Lambda(p_{\text{wait}})} \frac{R^{-1}(h, \underline{\Omega}) - C^{-1}(h)}{C^{-1}(h)} \quad (5)$$

where Ξ defines the ensemble of distributions such that the Belief Propagation (BP) decoder can converge on the desired range of capacities. In other words, we search in the class of output degree distributions that converge on the corresponding range of capacities and minimize the *rate overhead*, *i.e.* the gap between the coding rate R and the instantaneous capacity.

2) *Information Content evolution equations*: Raptor codes resemble LDPC codes, and the analysis is somewhat similar. Thus, in the same way that the optimization of LDPC codes boils down to the optimization of a left irregularity profile, the optimization of a Raptor code consists in optimizing the output degree distribution of the rateless part of the Raptor code, namely $\omega(x)$. This can be done with a technique called density evolution (DE) (see [8] for a detailed presentation). We recall one main result which is the equation that describe the evolution of the IC from one iteration to another, for a Raptor code with an LDPC precode and transfer function $T(\cdot)$, decoded on a BIAWGN of capacity $C(h)$:

$$\begin{aligned} x_u^{(l)} &= F(x_u^{(l-1)}, C(h), T(\cdot)), \\ &= 1 - \sum_{j=1}^{d_c} \omega_j J \left((j-1) J^{-1} \left(1 - \sum_{i=1}^{d_v} \iota_i \right. \right. \\ &\quad \left. \left. J((i-1) J^{-1}(x_u^{(l-1)}) + J^{-1}(T(x_{ext}^{(l-1)}))) \right) \right) \\ &\quad \left. + J^{-1}(1 - C(h)) \right). \end{aligned} \quad (6)$$

3) *Optimization of Raptor codes for quasi-static fading channels*: We formulate the optimization problem for a quasi-static fading channel. In this scenario, the encoder sees a channel with random capacity. Hence, Raptor codes that perform well on such channel must perform simultaneously well over different BIAWGN channels corresponding to a wide range of capacities. In order to guarantee the convergence of the decoder for such *range* of channel capacities $[C_0(p_{\text{wait}}), 1)$, we show how to write the IC evolution equations (6). Recently [8], we developed a method that enables to solve the constraint for one given channel capacity. We now rely on such results to constrain the optimization problem *simultaneously* for different channel capacities.

The equations of IC evolution are only valid for a given value of the average input symbol degree α , in the distribution $\iota(x)$ (see [8]). The α parameter is fixed in order to deal with a linear problem and it is chosen to minimize the overhead. This is related to the design of the output degree distribution on the corresponding BIAWGN channel. Moreover, this parameter represents the average degree of an input symbol and, as the output symbols are received the input symbols become more connected in the decoding Tanner graph. More precisely, for a given distribution $\Omega(x)$, the quantity α depends on the operating rate $R = \Omega'(1)/\alpha$, which can also be written as $\alpha = \alpha_0 R^{-1}$ where α_0 is the average degree when $R = 1$ (*i.e.* when the number of received output symbols equals the number of input symbols). If we want to guarantee convergence for different capacities, we have to account for the fact that the

average input degree α varies with the instantaneous operating rate. To this end, we make the assumption that the Raptor code operates close to the capacity, i.e. $R \approx C$. Consequently, given a capacity value C , the parameter α writes as $\alpha = \alpha_0 C^{-1}$, where α_0 becomes the design parameter. By discretizing the range of capacities $[C_0, 1)$ into $\{C_i\}$ the optimization problem is finally written as follows:

$$\omega_{opt}(x) = \arg \min_{\omega(x)} \sum_j \frac{\omega_j}{j}, \quad (7)$$

subject to the constraints:

$$\begin{aligned} [C_1] \quad & \sum_i \omega_i = 1, \\ [C_2] \quad & F(x, C_i) > x \quad \forall x \in [0; x_0 - \delta_C] \quad \text{for } C_i, \\ [C_3] \quad & F(0, C_i) > \varepsilon \quad \text{for some } \varepsilon > 0, \\ [C_4] \quad & F'(0, C_i) > 1. \end{aligned}$$

Similarly to the optimization for the BIAWGNC, it appears that there is an optimal value for the parameter α_0 , i.e. there is a value such that the cost function is minimized.

IV. RAPTOR CODES WITH IMPERFECT CSIR

Instantaneous channel knowledge at the receiver is necessary to compute the log-likelihood ratios (LLRs) and feed the iterative decoder. In the previous sections, we assumed that perfect channel knowledge is available at the receiver. However, this assumption is not longer valid in most of practical wireless systems. The main consequence of channel estimation errors is that perfect coherent demodulation is not possible any more, which can severely affect the system performance. This motivates us to study the design of optimal Raptor codes with imperfect channel estimation.

Channel estimation: The transmitter, before sending the data, can teach the channel to the receiver by sending a training sequence of N symbols $\mathbf{x}_T = (x_{T,1}, \dots, x_{T,N})^T$. For quasi-static fading, the coherence time of the channel is much longer than the training time. Moreover, we denote the average energy of the training symbols by $P_T = \frac{1}{N} \text{tr}(\mathbf{x}_T \mathbf{x}_T^\dagger)$. This sequence is affected by the channel gain H , allowing the receiver to perform ML estimation of H from the observed signals $\mathbf{y}_T = H\mathbf{x}_T + \mathbf{z}_T$ and \mathbf{x}_T . This yields to $\hat{H} = H + \mathcal{E}$, where \mathcal{E} denotes the estimation error of variance $\sigma_{\mathcal{E}}^2 = \text{SNR}_T^{-1}$ with $\text{SNR}_T = \frac{NP_T}{\sigma_Z^2}$. The receiver only knows the estimate \hat{H} and a characterization of its accuracy in terms of the conditional pdf $P_{H|\hat{H}}$. This pdf can be obtained by using the likelihood function, the pdf $W(y|x, H)$, and P_H and is given by:

$$P_{H|\hat{H}} = \mathcal{CN}(\delta\hat{H}, \delta\sigma_{\mathcal{E}}^2), \quad \text{with } \delta = \sigma_H^2 / (\sigma_H^2 + \sigma_{\mathcal{E}}^2). \quad (8)$$

In [9] the authors used the estimation accuracy given by (8) to derive a ML decoder that minimizes the average of the transmission error probability over all channel estimation errors. They also show that the mismatched ML decoder is not adapted to imperfect channel estimation. Furthermore, their decoder achieves the capacity of a composite channel.

These ideas will serve as the basis to solve the well-known problem of the mismatched LLR in Raptor codes. The

mismatched LLR simply consists in replacing the unknown channel gain by its estimate in the standard LLR expression. However, this operation does not lead to a consistent measure of information (see discussions in [10]). We first review the mismatched LLR to then derive a new LLR adapted to the channel estimation errors that leads to a consistent measure of information, which is an important assumption for characterizing the BP decoder under Gaussian approximation with IC evolution.

A. Mismatched LLRs

The mismatched log-likelihood ratios, which consists of replacing H by its estimate \hat{H} , are computed as follows:

$$L_M(y, \hat{h}) = \log \frac{W(Y_i = y | X_i = 1, H = \hat{h})}{W(Y_i = y | X_i = -1, H = \hat{h})} = \frac{4\hat{h}^\dagger y}{\sigma_Z^2}. \quad (9)$$

An LLR is said a consistent measure of information if its conditional density is *symmetric* [11]. For a QPSK modulation and a quasi-static fading channel, the symmetry condition implies that the variance $\text{Var}(L_M(y, \hat{h}) | X = 1, \hat{H} = \hat{h})$ is equal to $4 \mathbb{E}_{Y|X\hat{H}} \{L_M(y, \hat{h}) | X = 1, \hat{H} = \hat{h}\}$.

It is not difficult to compute these quantities:

$$\begin{aligned} \mathbb{E}_{Y|X\hat{H}} \{L_M(y, \hat{h}) | X = 1, \hat{H} = \hat{h}\} &= \frac{4\delta|\hat{h}|^2}{\sigma_Z^2}, \\ \text{Var}(L_M(y, \hat{h}) | X = 1, \hat{H} = \hat{h}) &= \frac{4^2 |\hat{h}|^2 (\sigma_Z^2 + \delta\sigma_{\mathcal{E}}^2)}{(\sigma_Z^2)^2}, \end{aligned} \quad (10)$$

which shows that in presence of imperfect channel estimation, the mismatched LLR does not lead to a consistent measure of information.

B. LLRs using channel estimation accuracy

We now adapt the LLRs to the channel estimation errors. To this end, we compute the log-likelihood ratio using the composite channel obtained by averaging the original channel $W(y|x, H)$, which depends on the unknown gain H , over the *a posteriori* pdf of H given \hat{H} . After some algebra [9], we obtain:

$$\widetilde{W}(y|x, \hat{H}) = \mathcal{CN}(\delta\hat{H}x, \sigma_Z^2 + \delta\sigma_{\mathcal{E}}^2|x^2), \quad (11)$$

where $\delta = \frac{\text{SNR}_T \sigma_H^2}{\text{SNR}_T \sigma_H^2 + 1}$. The averaged channel contains some additional noise related to the estimation errors. Actually, we can derive the LLR expression corresponding to the received bits via the composite channel:

$$\begin{aligned} L_{opt}(Y_i = y, \hat{h}) &= \log \frac{\widetilde{W}(Y_i = y | X_i = 1, \hat{H} = \hat{h})}{\widetilde{W}(Y_i = y | X_i = -1, \hat{H} = \hat{h})}, \\ &= \frac{4\delta\hat{h}^\dagger y}{\sigma_Z^2 + \delta\sigma_{\mathcal{E}}^2}. \end{aligned} \quad (12)$$

Hence, the variance and the mean of (12) satisfy:

$$\begin{aligned} \text{Var}(L_{opt}(y, \hat{h}) | X = 1, \hat{H} = \hat{h}) &= \frac{4^2 \delta^2 |\hat{h}|^2}{\sigma_Z^2 + \delta\sigma_{\mathcal{E}}^2} \\ &= 4 \mathbb{E}_{Y|X\hat{H}} \{L_{opt}(y, \hat{h}) | X = 1, \hat{H} = \hat{h}\}. \end{aligned} \quad (13)$$

which implies that (12) is Gaussian and a consistent measure of information.

V. SIMULATION RESULTS

In previous sections we showed that the optimization of an output degree distribution basically depends on the operating range of capacities. Suppose that a maximum delay is required, then outages will occur. The range of capacities of interest corresponds to the maximum delay, which is determined by the channel SNR and the outage probability tolerated.

In this section, we provide numerical results to analyze the performance of Raptor codes over quasi-static Rayleigh-fading channels under delay constraints. In particular, we optimized an output degree distribution for a quasi-static fading channel with SNR=12dB and outage probability $p_{\text{wait}} = 10^{-4}$. Fig 2 shows that this leads to a maximum delay $\Delta = 10$, or equivalently, a range of capacities equal to $[0.1, 1)$. We optimized an output degree distribution for this range and ran simulations. The overhead, defined as the gap to the capacity (right-hand side of equation (5)) is computed with IC evolution. The simulations reported on Fig. 2 show that, for the desired range of capacities, the optimized distribution operates within 10% of the instantaneous capacity.

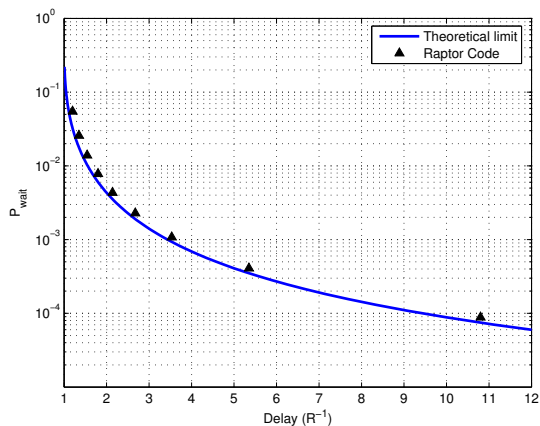


Fig. 2. Asymptotic performance, in terms of p_{wait} versus delay, of a Raptor codes optimized for a Rayleigh quasi-static fading channel (SNR=12dB) with perfect CSIR. Theoretical limits are also reported.

Fig 3 shows the performance of Raptor codes with finite size $K = 8192$, and 16-QAM input alphabet, over a quasi-static Rayleigh-fading channel, in the scenarios of: (i) imperfect CSIR by using the mismatched LLRs (expression (9)) and (ii) imperfect CSIR by using the LLRs with channel estimation accuracy (expression (12)). These results indicate that mismatched LLRs are sub-optimal for short training sequences, and confirmed the adequacy of the improved LLRs. This performance improvement was obtained without introducing any additional complexity.

VI. CONCLUSION

This paper studied the problem of rateless coding in practical wireless systems, where CSI is available at the receiver but not at the transmitter. We showed how Raptor codes can be optimized for communication over quasi-static fading

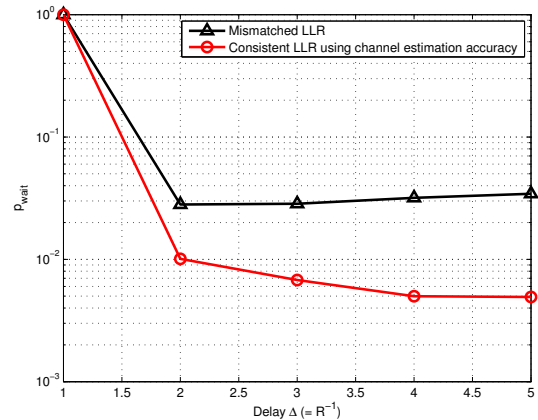


Fig. 3. Finite length performance of Raptor codes with imperfect channel estimation at the receiver ($N = 1$ training symbol and $P_T = P$).

channels under delay constraints. Simulation results showed that such codes efficiently adapt in an opportunistic manner to the communication conditions, *i.e.*, decoding in less time when the channel is good and taking more time to decode when the channel is bad. Moreover, we also studied the case where the receiver has only access to a noisy estimate of the channel. A characterization of channel estimation errors is used in an optimal manner to derive a consistent measure of information for decoding with imperfect CSI. As a perspective, it would be interesting to consider the framework proposed in [12], where each early successful decoding means that some extra channel uses become available for the next transmissions.

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