WEIGTHED SUM-RATE MAXIMIZATION IN MULTIUSER-OFDM SYSTEMS UNDER DIFFERENTIATED QUALITY-OF-SERVICE CONSTRAINTS

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ABSTRACT

We consider the maximization of the weighted sum-rate on a multiuser-OFDM downlink with adaptive modulation and power under a total transmit power constraint and a user-wise target BER. We allow each subcarrier to be shared by more than one user. We show that this optimization problem can be decomposed into two subproblems: a subcarrier assignment and a power allocation. We prove that the optimal subcarrier assignment is exclusive, that is each subcarrier is allocated to only one user. The optimal power allocation corresponds to a multilevel water-filling. When the achievable rate region is convex, the optimality of the exclusive subcarrier assignment for arbitrary weights means that the OFDMA is the optimal sharing scheme for various performance criteria.

1. INTRODUCTION

Much work has considered the problem of optimizing subcarrier and power assignment on a multi-user OFDM downlink by taking into account the channel state and the required QoS. Depending on the chosen performance criterion, different link adaptation strategies are obtained. It has often been assumed, as in [1–3], that subcarrier sharing is exclusive, that is only one user is allowed to transmit on a given subcarrier. This corresponds to orthogonal sharing (OFDMA) where no Multiple-Access Interference (MAI) is present. In [4], it has been proved that exclusive subcarrier sharing maximizes the sum-rate under the same target BER for all the users. In this paper, we prove that the exclusive subcarrier assignment, along with an appropriate power allocation, maximizes the weighted sum-rate in the case of user-wise target BER.

The optimization problem considered here is of special interest for multiple reasons. Firstly, compared to the sum-rate criterion, the weighted sum-rate permits to assign different priorities to the users by adjusting their respective weights accordingly. Secondly, taking into account the user-wise QoS constraint allows us to avoid a worst-case design where the lowest BER is guaranteed for all the users as in [4]. Finally, the optimality of exclusive subcarrier assignment can be generalized to other performance criteria.

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2. SYSTEM MODEL

Consider a single-cell OFDM downlink with N subcarriers and K users. With sufficiently-long cyclic prefix and perfect subcarrier synchronization, the discrete model is equivalent to N parallel flat fading channels with additive Gaussian noise. If we denote by $d_{k,n}$ the M-QAM symbol transmitted to the k^{th} user on the n^{th} subcarrier with an allocated power $p_{k,n}$, then the transmitted signal on that subcarrier is $x_n = \sum_{k=1}^{K} \sqrt{p_{k,n}} d_{k,n}$. This sum is the contributions of the K users sharing the considered subcarrier. The received signal corresponding to the k^{th} user on the n^{th} subcarrier is

$$y_{k,n} = \alpha_{k,n} \sum_{j=1}^{K} \sqrt{p_{j,n}} d_{j,n} + b_{k,n} \tag{1}$$

where $\alpha_{k,n}$ is the corresponding fading and $b_{k,n}$ is the additive noise. The k^{th} user needs to estimate the useful data symbol $d_{k,n}$ from $y_{k,n}$ for each n. Hence, we decompose the sum in (1) into a useful term and a MAI term as follows

$$y_{k,n} = \alpha_{k,n} \sqrt{p_{k,n}} d_{k,n} + (\alpha_{k,n} \sum_{j=1, j\neq k}^{K} \sqrt{p_{j,n}} d_{j,n} + b_{k,n}).$$

The average *Signal-to-Interference-plus-Noise Ratio* (SINR) $\gamma_{k,n}$ is the ratio of the useful-term average power to the average power of the remaining two terms (MAI+noise)

$$\gamma_{k,n} = \frac{E[|\alpha_{k,n} \sqrt{p_{k,n}} d_{k,n}|^2]}{E[|\alpha_{k,n} \sum_{j=1,j\neq k}^{K} \sqrt{p_{j,n}} d_{j,n} + b_{k,n}|^2]}$$

where $E[.]$ denotes the expectation with respect to $\{d_{k,n}\}$ and $\{b_{k,n}\}$ (the coefficients $\{\alpha_{k,n}\}$ are assumed perfectly known). With $E[|d_{k,n}|^2] = 1$ and $E[|b_{k,n}|^2] = \sigma_{k,n}^2$ we get

$$\gamma_{k,n} = \frac{|\alpha_{k,n}|^2 p_{k,n}}{|\alpha_{k,n}|^2 \sum_{j=1,j\neq k}^{K} p_{j,n} + \sigma_{k,n}^2}. \tag{2}$$

Under the assumption of Gaussian MAI (for a large number of users), we can use the the following BER approximation for M-QAM over Gaussian channels [5]

$$BER_k = 0.2 \exp\left(\frac{-1.5\gamma_{k,n}}{2^{q_{k,n}} - 1}\right) \tag{3}$$

where BER_k is the target BER for the k^{th} user and $q_{k,n}$ is the number of bits per M-QAM symbol. From (3) we get $q_{k,n} = \log_2 (1 + \gamma_{k,n}/\Gamma_k)$ with

$$\Gamma_k = -\ln(5BER_k)/1.5. \tag{4}$$

If we divide $q_{k,n}$ by the OFDM symbol duration T_s, we obtain the data rate for the k^{th} user on the n^{th} subcarrier. Assuming a coherent demodulation with a Nyquist matched filter, $1/T_s$ is equal to the subcarrier spacing ($1/T_s = B/N$) where B is the total OFDM bandwidth. Thus, the quantity $q_{k,n}$ is equivalent to the *spectral efficiency* in bps/Hz. However, $q_{k,n}$ is simply called "data rate" in what follows. So, the data rate achieved by the k^{th} user on the N subcarriers is

$$q_k(P) = \sum_{n=1}^{N} q_{k,n} = \sum_{n=1}^{N} \log_2 (1 + \gamma_{k,n}/\Gamma_k) \tag{5}$$

where P is the *Power Allocation Matrix* $P = [p_{k,n}]$. In the next section, we formulate the weighted sum-rate maximization problem and provide its solution.

3. MAXIMIZING THE WEIGHTED SUM-RATE

The *weighted sum-rate* is defined by $r(P) = \sum_{k=1}^{K} w_k \; q_k(P)$ where w_k is the weighting factor for the k^{th} user. From (5), this sum can be written as follows

$$r(P) = \sum_{k=1}^{K} w_k \sum_{n=1}^{N} \log_2 (1 + \gamma_{k,n}/\Gamma_k) \,. \tag{6}$$

We consider the following optimization problem

$$r^* = \max_P r(P) \tag{7}$$

subject to the total power constraint

$$\sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} = p_{tot}. \tag{8}$$

The link adaptation is reduced here to power allocation and the "subcarrier assignment" is implicit. In fact, when the power granted to a given user on a given subcarrier is non-null, we can say that this subcarrier is allocated to that user. But this allocation is not necessarily exclusive since other users may have non-null powers on the same subcarrier. The optimization problem (7-8) is complicated because of the MAI term in (2). In fact, when more power is granted to a given user on a given subcarrier, the data rate of this user increases, but simultaneously, more interference is added to the other users on the same subcarrier, lowering their achievable data rates. In what follows, we decompose this optimization problem into two tractable subproblems.

3.1. Problem decomposition

We start by rewriting (6) as follows

$$r(P) = \sum_{n=1}^{N} r_n(P) \tag{9}$$

where $r_n(P)$ represents the weighted sum-rate achieved by the K users on the same subcarrier of index n, i.e.

$$r_n(P) = \sum_{k=1}^{K} w_k \log_2 (1 + \gamma_{k,n}/\Gamma_k). \tag{10}$$

Now, we define a new matrix $C = [c_{k,n}]$ by

$$c_{k,n} = p_{k,n}/p_n \in [0, 1] \tag{11}$$

where p_n is the power allocated to the n^{th} subcarrier

$$p_n = \sum_{k=1}^{K} p_{k,n}. \tag{12}$$

The matrix C results from the power allocation matrix P by dividing the elements of each column in P by their sum. The total power constraint (8) becomes

$$\sum_{n=1}^{N} p_n = p_{tot}. \tag{13}$$

According to (11) and (12), for all $n \in \{1, .., N\}$ we have

$$\sum_{k=1}^{K} c_{k,n} = 1. \tag{14}$$

So, $c_{k,n}$ is the fraction of power allocated to the k^{th} user on the n^{th} subcarrier out of the total power p_n allocated to all the users on this subcarrier. It is a non-discrete indicator of the subcarrier assignment. This is because when $c_{k,n} = 0$, the k^{th} user does not make use of the n^{th} subcarrier and, in the opposite case where $c_{k,n} = 1$, the considered subcarrier is exclusively allocated to this user. In all other "non-extreme" cases, the subcarrier is actually shared by more than one user. Therefore, we call C the *Subcarrier Assignment Matrix*.

Using (11) and (12), the SINR expression (2) becomes

$$\gamma_{k,n} = \frac{p_{k,n}}{p_n - p_{k,n} + \frac{\sigma_{k,n}^2}{|\alpha_{k,n}|^2}} = \frac{c_{k,n} p_n}{(1 - c_{k,n}) p_n + \frac{\sigma_{k,n}^2}{|\alpha_{k,n}|^2}}. \tag{15}$$

By substituting (15) into (10), we see that $r_n(P)$ is a function of the power p_n and of $\mathbf{c_n} = [c_{1,n}, c_{2,n}, .., c_{K,n}]^t$, which is the n^{th} column in the matrix C, that is

$$r_n(P) = r_n(p_n, \mathbf{c_n}) = \sum_{k=1}^{K} w_k \log_2 \left[1 + \frac{c_{k,n} p_n}{\left[(1 - c_{k,n}) p_n + \frac{\sigma_{k,n}^2}{|\alpha_{k,n}|^2} \right] \Gamma_k} \right] \tag{16}$$

Now, the weighted sum-rate $r(P)$ in (9) can be seen as a function of the vector $\mathbf{p} = [p_1, p_2, .., p_N]$, that describes how p_{tot} is divided among the subcarriers, and of the matrix C, that is $r(\mathbf{p}, C) = \sum_{n=1}^{N} r_n(p_n, \mathbf{c_n})$. So, from (7) we get

$$r^* = \max_{\mathbf{p},C} r(\mathbf{p}, C) \tag{17}$$

subject to (13) and (14) (instead of (8)). Furthermore, without any additional assumptions on the function $r(\mathbf{p}, C)$, we can carry out the joint optimization in (17) as follows

$$r^* = \max_{\mathbf{p}} \max_{C} \sum_{n=1}^{N} r_n(p_n, \mathbf{c_n}). \tag{18}$$

Notice that (14) is a set of N independent constraints on the columns $\{\mathbf{c_n}\}$ of C. This means that, for instance, the way the first-carrier allocated power p_1 is divided among the users has no effect on the achieved data rate on the other subcarriers. Thus, equation (18) can be written as follows

$$r^* = \max_{\mathbf{p}} \sum_{n=1}^{N} \max_{\mathbf{c_n}} r_n(p_n, \mathbf{c_n}). \tag{19}$$

This last equation shows that the maximum weighted sum-rate can be reached by first finding the optimal power partitioning $\mathbf{c}_\mathbf{n}^*$, that maximizes the data rate $r_n(p_n, \mathbf{c_n})$ on each subcarrier with a given p_n, and then finding the power allocation $\mathbf{p}^* = [p_1^*, .., p_n^*]$ that maximizes the overall weighted sum-rate given by $\sum_{n=1}^{N} r_n(p_n, \mathbf{c}_\mathbf{n}^*)$. So, the problem (19) can be decomposed into the following two subproblems

$$\mathbf{c}_\mathbf{n}^*(p_n) = arg \max_{\mathbf{c_n}} r_n(p_n, \mathbf{c_n}) \;\; s.t. \;\; (14), \tag{20}$$

$$\mathbf{p}^* = arg \max_{\mathbf{p}} \sum_{n=1}^{N} r_n(p_n, \mathbf{c}_\mathbf{n}^*(p_n)) \;\; s.t. \;\; (13). \tag{21}$$

Remember that an argument was given above for calling C the “subcarrier assignment matrix”. Thus, the next section that treats the first subproblem (20) is called the “Optimal subcarrier assignment”.

3.2. Optimal subcarrier assignment

The following theorem provides the solution to the first optimization subproblem (20).

Theorem 1 *The weighted sum-rate $r_n(p_n, \mathbf{c_n})$, given in (16), achieved by the K users sharing an amount of power p_n on the same subcarrier of index n, is maximized when the whole power p_n is allocated to the user of index*

$$k_n^*(p_n) = arg \max_{k=1,..,K} \left(1 + \frac{|\alpha_{k,n}|^2 p_n}{\sigma_{k,n}^2 \Gamma_k}\right)^{w_k}. \tag{22}$$

So, the optimal vector $\mathbf{c}_\mathbf{n}^*$ has only one non-null component equal to one at the position $k_n^*(p_n)$. Equivalently, we have

$$\max_{\mathbf{c_n}} r_n(p_n, \mathbf{c_n}) = \max_{k=1,..,K} w_k \log_2 \left(1 + \frac{|\alpha_{k,n}|^2 p_n}{\sigma_{k,n}^2 \Gamma_k}\right). \tag{23}$$

We provide only the outline of the proof that uses mathematical induction with respect to K. Using (14) and (16), the quantity to be maximized in (23) becomes a function f_K of the $K-1$ variables $c_{1,n}, .., c_{K-1,n}$ as follows

$$\begin{aligned} f_K(c_{1,n}, .., c_{K-1,n}) = \\ \sum_{k=1}^{K-1} w_k \log_2 \left[1 + \frac{c_{k,n} p_n}{\left((1 - c_{k,n}) p_n + \frac{\sigma_{k,n}^2}{|\alpha_{k,n}|^2}\right) \Gamma_k}\right] \\ + w_K \log_2 \left[1 + \frac{(1 - \sum_{j=1}^{K-1} c_{j,n}) p_n}{\left((\sum_{j=1}^{K-1} c_{j,n}) p_n + \frac{\sigma_{K,n}^2}{|\alpha_{K,n}|^2}\right) \Gamma_k}\right]. \end{aligned}$$

By showing that $f_2(c_{1,n})$ is strictly convex on $]0, 1[$, we prove that its maximum corresponds to $c_{1,n} = 0$ or $c_{1,n} = 1$. This proves (23) for $K = 2$. Now, we assume that (23) is satisfied up to K users and we prove it for $K + 1$. The function f_{K+1} represents a hyper-surface in a K-dimension hyper-space. By showing that its hessian is a positive definite matrix on the open set $\Omega = \{(c_{1,n}, .., c_{K,n}) \in]0, 1[^K : \sum_{k=1}^{k=K} c_{k,n} < 1\}$, we conclude that this hyper-surface is strictly convex on Ω. This means that its maximum lies on the boundary of the closed set $\{(c_{1,n}, .., c_{K,n}) \in [0, 1]^K : \sum_{k=1}^{k=K} c_{k,n} \leq 1\}$. But this boundary consists of K parts contained respectively in K hyper-plans $H_1, .., H_K$ with $H_k = \{(c_{1,n}, .., c_{K,n}) \in [0, 1]^K : c_{k,n} = 0\}$. Each part is in its turn a hyper-surface of dimension $K - 1$ representing a function of the form f_K defined above. So, the induction hypothesis can be used to show that each one of these functions f_K is maximized when its arguments are all null except for one argument $c_{k_n^*,n} = 1$ whose index k_n^* is obviously that of the optimal user.

3.3. Optimal power allocation

From (16), the optimal subcarrier assignment $\mathbf{c}_\mathbf{n}^*$ given by *Theorem 1* yields on the n^{th} subcarrier a weighted sum-rate equal to

$$r_n(p_n, \mathbf{c}_\mathbf{n}^*(p_n)) = w_{k_n^*(p_n)} \log_2 \left(1 + \frac{|\alpha_{k_n^*(p_n),n}|^2 p_n}{\sigma_{k_n^*(p_n),n}^2 \Gamma_k}\right)$$

where $k_n^*(p_n)$ is given by (22). So, the problem (21) becomes

$$r^* = \max_{p_1,..,p_N} \sum_{n=1}^{N} r_n(p_n, \mathbf{c}_\mathbf{n}^*(p_n)). \tag{24}$$

Unlike the case of equal weights where the power allocation step is resolved by simple water-filling, the optimization problem (24) is a harder one. This is due to the dependency of the

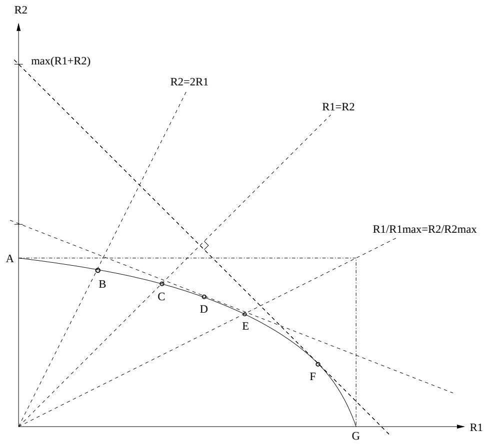

Fig. 1. Achievable rate region and optimal points for various performance measures (convex region case),
A, G=user 2,1 maximum single-user rate $R_{2,max}, R_{1,max}$,
B=maximum proportional rates $max\ R_1$ s.t. $R_2 = 2R_1$,
C=maximum common rate (max-min) $max\ min\{R_1, R_2\}$,
D=maximum weighted-rate $max(\frac{1}{3}R_1 + \frac{2}{3}R_2)$,
E=maximum balanced rates $max\ R_1$ s.t. $\frac{R_1}{R_{1,max}} = \frac{R_2}{R_{2,max}}$,
F=maximum sum-rate $max(R_1 + R_2)$.

optimal scheduled user (22) on the power p_n. However, for a given scheduling $(k_1, .., k_N)$, where k_n is the index of the scheduled user on the n^{th} subcarrier, the following sum

$$r = \sum_{n=1}^{N} w_{k_n} \log_2 \left(1 + \frac{|\alpha_{k_n,n}|^2 p_n}{\sigma^2_{k_n,n}\Gamma_k}\right) \tag{25}$$

is maximized with respect to the powers by *multilevel water-filling* as follows $p_n = \max\left\{0, \frac{w_{k_n}}{v} - \frac{\sigma^2_{k_n,n}\Gamma_{k_n}}{|\alpha_{k_n,n}|^2}\right\}$. The parameter v can be calculated from the power constraint (13). This result can be proved by the Lagrange-multipliers method. The optimality of water-filling power allocation is true in particular for the optimal scheduling $(k_1^*, .., k_N^*)$. So, the optimal scheduling can be found by an exhaustive search over the K^N possible ones. For each candidate $(k_1, .., k_N)$, we use the multilevel water-filling to calculate the corresponding powers and (25) to find the resulting weighted sum-rate. The optimal solution corresponds to the maximum weighted sum-rate. Such exhaustive search is certainly unpractical because of its high complexity. Thus, we propose hereafter a suboptimal solution based on equal powers.

3.4. Equal-powers suboptimal solution

When the users undergo i.i.d. fadings on the different subcarriers, the powers allocated to the different subcarriers have obviously the same statistical average. Thus, a subcarrier assignment based on equal powers $p_n = p_{tot}/N$ has a close-to-optimal average weighted sum-rate. With equal powers, equation (22) becomes $\tilde{k}_n = arg\max_k \left(1 + \frac{|\alpha_{k,n}|^2\ p_{tot}}{\sigma^2_{k,n}\Gamma_k\ N}\right)^{w_k}$. So, the suboptimal scheduling $(\tilde{k}_1, .., \tilde{k}_N)$ can be found with low complexity since no exhaustive search is needed.

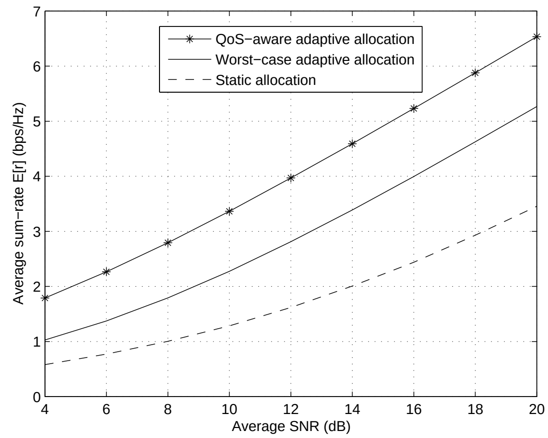

Fig. 2. Average sum-rate versus average SNR for static and adaptive allocation schemes (Worst-case,QoS-aware) ($K = 8$ users, $N = 256$ subcarriers, $BER = 10^{-3}, 10^{-5}$, $P_{tot} = 1$).

4. GENERALIZATION OF EXCLUSIVE SUBCARRIER ASSIGNMENT OPTIMALITY

We want to generalize the optimality of the exclusive subcarrier assignment to performance criteria other than the weighted sum-rate. We know that regardless of the performance criterion (sum-rate [4], max-min rate [2], balanced rates [6] or proportional rates [3]), the optimal rates correspond to some point on the boundary of the rate region (Fig. 1). When the rate region is convex, each point on its boundary is also the optimal solution that maximizes the weighted sum-rate for some weighting vector. So, only in the convex case we obtain that the exclusive subcarrier-assignment, which we proved to be optimal in terms of the weighted sum-rate, is the optimal subcarrier sharing whatever the performance criterion is. In [7], the authors showed that the rate region is convex for exclusive subcarrier assignment and infinite number of subcarriers. Using this result along with our *Theorem 1* above, we can say that any non-exclusive subcarrier sharing leads to some point inside the rate region boundary which is approximately convex for a large number of subcarriers.

5. SIMULATION RESULTS

Consider i.i.d. Rayleigh fadings with $E[|\alpha_{k,n}|^2] = 1$. We assume that $\sigma^2_{k,n}$= $\sigma^2 = N_0\frac{B}{N}$ where N_0 is the noise power spectral density and B is the total bandwidth. We define an "average SNR" by $\bar{\gamma} = p_{tot}/(N_0B) = p_{tot}/(N\sigma^2)$. Firstly, we consider the case of $N = 256$ subcarriers and $K = 8$ users belonging to two classes of service $BER = 10^{-2}$ and 10^{-5}. We assume equal weights in order to compare our QoS-aware solution to the worst-case one proposed in [4]. From (22), the user to be scheduled on each subcarrier with equal weights is simply given by $k_n^* = arg\max_k |\alpha_{k,n}|^2/(\sigma_{k,n}^2 \Gamma_k)$. In this case, the subcarrier assignment becomes independent of the power allocation. In Fig. 2, we plot the average sum-rate versus $\bar{\gamma}$ for three different resource allocation schemes. The dashed curve corresponds to a static subcarrier assignment, where $N/K = 256/8 = 32$ subcarriers are allocated to each user, followed by a water-filling of power. The remaining solid-line curves correspond to adaptive subcarrier and power allocation. We mean by "worst case" that the resources are allocated such that the lowest BER (10^{-5}) is guaranteed to all the users regardless of their required QoS levels. In the "QoS-Aware" case, the required BER's are just scrupulously satisfied. These curves show the advantage of the adaptive allocation over the static one. Moreover, our proposed QoS-aware allocation yields a significant enhancement compared to the worst-case one.

Now consider the case of two users with unequal weights $(w_1, w_2) = (0.6, 0.4)$ and $N = 8$ subcarriers. The value of N is kept small in order to have bounded simulation time when exhaustive search is implemented. In Fig. 3, we show the average weighted sum-rate versus the average SNR for both the optimal solution, found by exhaustive search, and the suboptimal one based on equal powers. As expected, there is no significant performance loss when the water-filling is replaced by equal powers. On the same figure we also compare the suboptimal adaptive subcarrier assignment to a static subcarrier assignment where a fixed N_1 (resp. N_2) subcarriers are allocated to the first (resp. the second) user with $N_1/N_2 \simeq w_1/w_2$. We see that a significant gain is achieved by adaptive subcarrier allocation even when this allocation is based on equal powers. This gain is due to the multiuser diversity exploited on each subcarrier when the scheduled user is dynamically chosen.

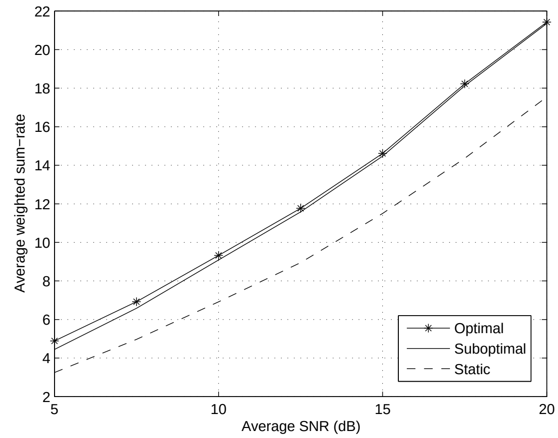

Fig. 3. Average weighted sum-rate versus average SNR for optimal, suboptimal and static schemes ($K = 2$ users, $N = 8$ subcarriers, $w_1 = 0.6$, $w_2 = 0.4$, $BER = 10^{-3}$, $P_{tot} = 1$).

6. CONCLUSION

We considered the problem of maximizing the weighted sum-rate on a multiuser OFDM downlink under a maximum transmit power and a user-wise BER constraint where simultaneous sharing of each subcarrier by multiple users is allowed. We decomposed this problem into two tractable subproblems corresponding to subcarrier assignment and power allocation. Resolving the first subproblem showed that the optimal solution is an exclusive subcarrier assignment. Regarding the second subproblem, the optimal power partitioning is obtained by a multilevel water-filling. Since the solutions to these subproblems are inter-dependent, an exhaustive search is needed to find the optimal user scheduling. Therefore, we proposed a suboptimal strategy based on equal powers to avoid the burden of exhaustive search. Finally, we extended the optimality of exclusive subcarrier assignment to other performance criteria such as the max-min rate and the proportional rates. Simulations results showed that the proposed solutions yield significant gain in terms of weighted sum-rate compared to static subcarrier allocation.

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