Optimal Subcarrier Sharing for Weighted Sum of Rates Maximization in Multiuser-OFDM Systems

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***Abstract*— We consider the maximization of the weighted sum of rates in a multiuser OFDM system with dynamic subcarrier allocation and subcarrier-wise adaptive modulation and power. We assume a total peak power constraint and a target Bit Error Rate (BER). We allow each subcarrier to be shared by more than one user. This sharing implies an interference which the receivers deal with as noise. In this paper, we show that the resulting non-convex optimization problem can be decomposed into two tractable subproblems: a subcarrier assignment and a power allocation. We prove that the optimal subcarrier assignment is exclusive, i.e. each subcarrier has to be allocated to one user only. The optimal power allocation corresponds to a multilevel water-filling. Since the solutions to these two subproblems are mutually dependent, one can use exhaustive search to explicit the optimal subcarrier assignment. To avoid the burden of exhaustive search, we show that a close-to-optimal low-complexity solution is obtained if the total power is equally partitioned among the subcarriers. Finally, simulation results show that the proposed suboptimal solution yields a significant gain in terms of average weighted sum of rates compared to a traditional static subcarrier allocation.**[1]

I. Introduction

Orthogonal-Frequency Division Multiplexing (OFDM) [1] is an attractive bandwidth-efficient modulation technique for broadband services in dispersive environments. The choice of OFDM in key standards, such as IEEE802.11a/g [2] for WLAN and IEEE802.16d (WiMax) [3], leads to an increasing interest in enhancing the performance of multiuser OFDM systems. Performance improvement can be achieved by optimizing the way the different parameters, such as the subcarrier-wise modulation and power, are adapted to the channel conditions and to the required Quality of Service (QoS). This yields different link adaptation strategies depending on the chosen optimality criterion [5]–[9].

In this paper, we consider the weighted sum of instantaneous data rates on an uncoded OFDM downlink under a total peak power constraint and a target BER. The weighted sum of rates is a relevant criterion for heavy download applications when the users are assigned to different priority levels. Priorities are controlled by adjusting the weighting factors. Moreover, maximizing the weighted sum of rates for arbitrary weights characterizes the boundary of the achievable rate region. By considering the uncoded case with M-QAM modulation and realistic BER values, our approach is more connected to real systems than those dealing with error-free capacities where the transmit alphabet and the decoding complexity are unconstrained [6], [9].

In our formulation, we allow each subcarrier to be shared by more than one user simultaneously without the use of hierarchical modulation. For each user, sharing a given subcarrier with other users results in an interference which is considered as noise. This interference makes the allocation problem a complicated one. In fact, increasing a given user data rate by granting more power adds more interference to the other users sharing the same subcarrier and, consequently, their data rates are degraded. However, we show how the resulting non-convex optimization problem can be decomposed into two tractable subproblems: a subcarrier assignment and a power allocation. Resolving the first subproblem reveals that the optimal subcarrier sharing is *exclusive*, that is each subcarrier must be allocated to one user only. This corresponds to the *Orthogonal-Frequency Division Multiple-Access* (OFDMA) scheme. OFDMA was adopted in WiMax [4] with a special support for adaptive subcarrier and power allocation. A similar result about OFDMA optimality was stated in [7] in the special case of equal weights where the problem reduces to a total rate maximization.

Unfortunately, the solutions to the above subproblems are mutually dependent. This means that the choice of the scheduled user on a given subcarrier requires the knowledge of the power allocated to that subcarrier. The optimal subcarrier assignment can be found by exhaustive search. Once the optimal subcarrier assignment is found, the solution to the second subproblem is simply a *multilevel water-filling* [9] of the total power.

To avoid the complexity of exhaustive search, we propose a suboptimal solution by partitioning the total power equally among the subcarriers. Simulation results show that this low-complexity optimal solution has a close-to-optimal performance in terms of average weighted sum of rates. Furthermore, due to the multiuser diversity gain, both optimal and sub-optimal solutions outperform the static subcarrier allocation where a fixed set of subcarriers is allocated to each user. This diversity gain follows from the fact that a "bad" subcarrier, that suffers from a deep fade, for one user may be a "good" subcarrier for another user.

[1] *This work was supported by a French ANR RNRT-Grant under project DIVINE.*

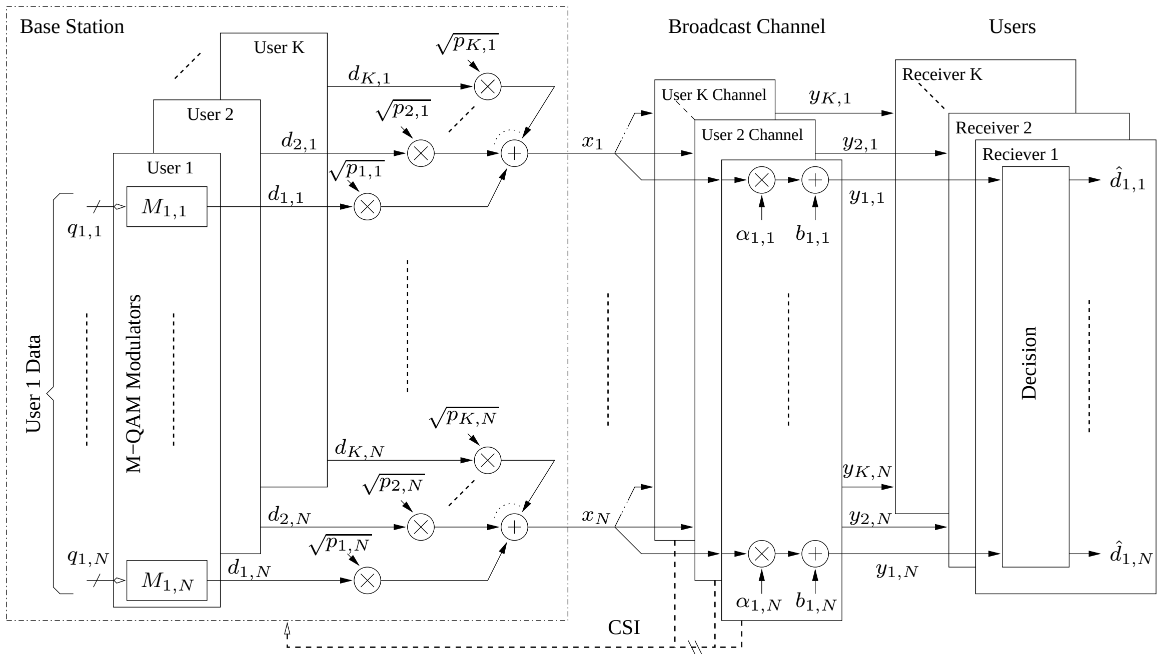

Fig. 1. System Model.

II. System model

Consider a single-cell OFDM downlink with N subcarriers and K users. With sufficiently long *cyclic prefix* and perfect subcarrier synchronization, the discrete model of the frequency-selective fading channel is equivalent to N parallel flat-fading channels with additive Gaussian noise (Fig. 1.). If we denote by $d_{k,n}$ the M-QAM symbol transmitted to the k^{th} user on the n^{th} subcarrier with an allocated power $p_{k,n}$, then the transmitted signal on that subcarrier is $x_n = \sum_{k=1}^{K} \sqrt{p_{k,n}} d_{k,n}$. This sum is the contributions of the K users sharing the considered subcarrier. The received signal corresponding to the k^{th} user on the n^{th} subcarrier is

$$y_{k,n} = \alpha_{k,n} \sum_{j=1}^{K} \sqrt{p_{j,n}} d_{j,n} + b_{k,n} \tag{1}$$

where $\alpha_{k,n}$ is the corresponding fading and $b_{k,n}$ is the additive noise. For slowly-fading channels, the coefficients $\{\alpha_{k,n}\}$ are considered invariant during one resource adaptation cycle. Moreover, they are assumed perfectly and instantaneously known to the base station.

The k^{th} user needs to estimate the useful data symbol $d_{k,n}$ from $y_{k,n}$ for each n. Hence, we decompose (1) into a useful term and an interference-plus-noise term as follows

$$y_{k,n} = \alpha_{k,n} \sqrt{p_{k,n}} d_{k,n} + (\alpha_{k,n} \sum_{j=1, j\neq k}^{K} \sqrt{p_{j,n}} d_{j,n} + b_{k,n}).$$

We define the average *Signal-to-Interference-plus-Noise Ratio* (SINR) by

$$\gamma_{k,n} = \frac{E[|\alpha_{k,n}\sqrt{p_{k,n}} d_{k,n}|^2]}{E[|\alpha_{k,n} \sum_{j=1,j\neq k}^{K} \sqrt{p_{j,n}} d_{j,n} + b_{k,n}|^2]}$$

where $E[.]$ denotes the expectation with respect to $\{d_{k,n}\}$ and $\{b_{k,n}\}$. With $E[|d_{k,n}|^2] = 1$ and $E[|b_{k,n}|^2] = \sigma_{k,n}^2$ we get

$$\gamma_{k,n} = \frac{|\alpha_{k,n}|^2 p_{k,n}}{|\alpha_{k,n}|^2 \sum_{j=1,j\neq k}^{K} p_{j,n} + \sigma_{k,n}^2}. \tag{2}$$

For large number of users, the interference term in (2) can be approximated by a Gaussian noise. In this case, we can use the following BER approximation from [11] for M-QAM signaling over Gaussian channels

$$BER = 0.2 \exp\left(\frac{-1.5\gamma_{k,n}}{2^{q_{k,n}} - 1}\right), \tag{3}$$

where BER denotes the target bit error rate for all the users, $\gamma_{k,n}$ is the average SINR given by (2) and $q_{k,n}$ is the number of bits per M-QAM symbol for the k^{th} user on the n^{th} subcarrier. From (3) we get $q_{k,n} = \log_2(1 + \gamma_{k,n}/\Gamma)$ where the BER-dependent parameter Γ, called the "coding gap" [11], is given by

$$\Gamma = -\ln(5BER)/1.5. \tag{4}$$

Note that if we divide $q_{k,n}$ by the OFDM symbol duration T_s, we obtain the data rate for the k^{th} user on the n^{th} subcarrier. Assuming a coherent demodulation with a Nyquist matched filter, $1/T_s$ is equal to the subcarrier spacing $(1/T_s = B/N)$ where B is the total OFDM bandwidth. Thus, the quantity $q_{k,n}$ is equivalent to the *spectral efficiency* in bps/Hz. However, $q_{k,n}$ is simply called "data rate" henceforth. The data rate achieved by the k^{th} user on the N subcarriers is

$$q_k(P) = \sum_{n=1}^{N} q_{k,n} = \sum_{n=1}^{N} \log_2(1 + \gamma_{k,n}/\Gamma), \tag{5}$$

where P is the *Power Allocation Matrix* $P = [p_{k,n}]$.

In the next section, we formulate the weighted sum of rates maximization problem and we decompose it into two tractable subproblems.

III. Maximizing the weighted sum of rates

The *Weighted Sum of Rates* is defined by

$$r(P) = \sum_{k=1}^{K} w_k \; q_k(P),$$

where w_k is the weighting factor for the k^{th} user. From (5), this sum can be written as follows

$$r(P) = \sum_{k=1}^{K} w_k \sum_{n=1}^{N} \log_2 \left(1 + \gamma_{k,n}/\Gamma\right). \tag{6}$$

Without loss of generality, we can assume that the weighting factors $\{w_k\}$ satisfy

$$\sum_{k=1}^{K} w_k = 1.$$

We consider the following optimization problem

$$r^* = \max_{P} r(P), \tag{7}$$

subject to the *total power constraint*

$$\sum_{n=1}^{N} \sum_{k=1}^{K} p_{k,n} = p_{tot}. \tag{8}$$

Thus, our objective is to find the appropriate power allocation that yields the maximum achievable data rate r^*. The BER constraint is taken into account in (6) through the parameter Γ defined in (4). The link adaptation is reduced here to power allocation. The term “subcarrier assignment” does not appear yet since it is implicitly performed via the power allocation. In fact, when the power granted to a given user on a given subcarrier is non-null, we can say that this subcarrier is allocated to that user. But this allocation is not necessarily exclusive since other users may also have non-null powers on the same subcarrier.

The optimization problem (7-8) is complicated because of the interference term in the SINR expression (2). In fact, when more power is granted to a given user on a given subcarrier, the data rate of this user increases. Meanwhile, more interference is added to the other users on the same subcarrier and, consequently, their achievable data rates are degraded. However, in what follows, we show how this non-convex optimization problem can be simplified by decomposing it into two tractable subproblems.

A. Problem decomposition

We start by rewriting (6) as follows

$$r(P) = \sum_{n=1}^{N} r_n(P), \tag{9}$$

where $r_n(P)$ represents the weighted sum of rates achieved by the K users on the same subcarrier of index n, i.e.

$$r_n(P) = \sum_{k=1}^{K} w_k \log_2 \left(1 + \gamma_{k,n}/\Gamma\right). \tag{10}$$

Now, we introduce a new matrix $C = [c_{k,n}]$ whose elements are defined by

$$c_{k,n} = p_{k,n}/p_n \in [0, 1], \tag{11}$$

where p_n is the power allocated to the n^{th} subcarrier

$$p_n = \sum_{k=1}^{K} p_{k,n}. \tag{12}$$

The matrix C results from the power allocation matrix P by dividing the elements of each column in P by their sum. The total power constraint (8) becomes

$$\sum_{n=1}^{N} p_n = p_{tot}. \tag{13}$$

According to (11) and (12), for all $n \in \{1, .., N\}$, the elements of the matrix C satisfy

$$\sum_{k=1}^{K} c_{k,n} = 1. \tag{14}$$

Thus, $c_{k,n}$ is the fraction of power allocated to the k^{th} user on the n^{th} subcarrier out of the total power p_n allocated to all the users on this subcarrier. The quantity $c_{k,n}$ can be considered as a non-discrete indicator of the subcarrier assignment. This is because when $c_{k,n} = 0$, the k^{th} user does not make use of the n^{th} subcarrier and, in the opposite case where $c_{k,n} = 1$, the considered subcarrier is exclusively allocated to this user. In all other “non-extreme” cases, the subcarrier is actually shared by more than one user. For this reason, we call C the *Subcarrier Assignment Matrix*.

Using (11) and (12), the SINR expression (2) becomes

$$\gamma_{k,n} = \frac{p_{k,n}}{p_n - p_{k,n} + \frac{\sigma^2_{k,n}}{|\alpha_{k,n}|^2}} = \frac{c_{k,n} p_n}{(1 - c_{k,n}) p_n + \frac{\sigma^2_{k,n}}{|\alpha_{k,n}|^2}}. \tag{15}$$

By substituting (15) into (10), we see that $r_n(P)$ is a function of the power p_n and of $\mathbf{c_n} = [c_{1,n}, c_{2,n}, .., c_{K,n}]^t$, which is the n^{th} column in the matrix C, i.e.

$$\begin{aligned} r_n(P) &= r_n(p_n, \mathbf{c_n}) \\ &= \sum_{k=1}^{K} w_k \log_2 \left[1 + \frac{c_{k,n} p_n}{\left((1 - c_{k,n}) p_n + \frac{\sigma^2_{k,n}}{|\alpha_{k,n}|^2}\right)\Gamma}\right]. \end{aligned} \tag{16}$$

It follows that the weighted sum of rates $r(P)$ in (9) can be seen as a function of the vector $\mathbf{p} = [p_1, p_2, .., p_N]$, that describes how the total power p_{tot} is divided among the N subcarriers, and of the matrix C. Thus, equation (9) becomes

$$r(\mathbf{p}, C) = \sum_{n=1}^{N} r_n(p_n, \mathbf{c_n}).$$

This allows us to transform the optimization problem (7) into the following equivalent form

$$r^* = \max_{\mathbf{p},C} r(\mathbf{p}, C), \tag{17}$$

subject to the constraints (13) and (14) which replace the total power constraint (8). Furthermore, without any additional assumptions on the function $r(\mathbf{p}, C)$, we can carry out the joint optimization in (17) through two consecutive steps as follows

$$r^* = \max_{\mathbf{p}} \left(\max_{C} \sum_{n=1}^{N} r_n(p_n, \mathbf{c_n}) \right). \tag{18}$$

Notice that (14) is a set of N independent constraints on the level of the individual columns $\mathbf{c_n}$'s of the matrix C. This means that, for instance, the way the first-carrier allocated power p_1 is divided among the users has no effect on the achieved data rate on the other subcarriers. Thus, equation (18) is equivalent to

$$r^* = \max_{\mathbf{p}} \sum_{n=1}^{N} \max_{\mathbf{c_n}} r_n(p_n, \mathbf{c_n}). \tag{19}$$

This last equation shows that the maximum weighted sum of rates can be reached by first finding the optimal power partitioning $\mathbf{c_n^*}$ that maximizes the data rate $r_n(p_n, \mathbf{c_n})$ for each subcarrier with a given p_n, and then finding the power allocation $\mathbf{p}^* = [p_1^*, .., p_n^*]$ that maximizes the overall weighted sum of rates $\sum_{n=1}^{N} r_n(p_n, \mathbf{c_n^*})$. Thus, the optimization problem (19) can be decomposed into the following subproblems

$$\mathbf{c_n^*}(p_n) = \arg\max_{\mathbf{c_n}} r_n(p_n, \mathbf{c_n}) \quad \text{subject to constraint } (14), \tag{20}$$

$$\mathbf{p}^* = \arg\max_{\mathbf{p}} \sum_{n=1}^{N} r_n(p_n, \mathbf{c_n^*}(p_n)) \quad \text{subject to constraint } (13). \tag{21}$$

In (20), the notation $\mathbf{c_n^*}(p_n)$ suggests that the optimal subcarrier assignment $\mathbf{c_n^*}$ on the n^{th} subcarrier may depend on the amount of power p_n allocated to that subcarrier.

Remember that an argument was given above for calling C the "subcarrier assignment matrix". Therefore, the following section that considers the first subproblem (20) is called "Optimal subcarrier assignment".

B. Optimal subcarrier assignment

Here we provide the solution to the first optimization subproblem (20). The following theorem shows that $r_n(p_n, \mathbf{c_n})$, defined in (16), is maximized when the whole power p_n is exclusively allocated to one user.

Theorem 1: The weighted sum of rates $r_n(p_n, \mathbf{c_n})$ achieved by the K users sharing a total amount of power p_n on the same subcarrier of index n is maximized when the whole power p_n is allocated to the user of index

$$k_n^*(p_n) = \arg\max_{k=1,..,K} \left(1 + \frac{|\alpha_{k,n}|^2 p_n}{\sigma_{k,n}^2 \Gamma}\right)^{w_k}. \tag{22}$$

In other words, the optimal vector $\mathbf{c_n^*}$ has only one non-null component equal to one at the position $k_n^*(p_n)$. Equivalently, we can write

$$\max_{\mathbf{c_n}} r_n(p_n, \mathbf{c_n}) = \max_{k=1,..,K} w_k \log_2 \left(1 + \frac{|\alpha_{k,n}|^2 p_n}{\sigma_{k,n}^2 \Gamma}\right). \tag{23}$$

Proof: The proof is carried out by mathematical induction with respect to the number of users K. Using (14) and (16), the quantity $r_n(p_n, \mathbf{c_n})$ to be maximized in (23) becomes a function f_K of the $K-1$ variables $c_{1,n}, .., c_{K-1,n}$ as follows

$$\begin{aligned} f_K(c_{1,n}, .., c_{K-1,n}) = \\ \sum_{k=1}^{K-1} w_k \log_2 \left[1 + \frac{c_{k,n} p_n}{\left((1 - c_{k,n}) p_n + \frac{\sigma_{k,n}^2}{|\alpha_{k,n}|^2}\right)\Gamma}\right] \\ + w_K \log_2 \left[1 + \frac{(1 - \sum_{j=1}^{K-1} c_{j,n}) p_n}{\left((\sum_{j=1}^{K-1} c_{j,n}) p_n + \frac{\sigma_{K,n}^2}{|\alpha_{K,n}|^2}\right)\Gamma}\right]. \end{aligned}$$

For $K = 2$ users, we get a function f_2 of one variable $c_{1,n}$. By showing that f_2 is strictly convex on $]0, 1[$, we prove that its maximum corresponds to $c_{1,n} = 0$ or/and $c_{1,n} = 1$. This proves (23) for $K = 2$. Then, one can assume that (23) is met up to K users and prove it for $K + 1$. ■

In high-SNR regime ($|\alpha_{k,n}|^2 p_n/(\sigma_{k,n}^2 \Gamma) \gg 1$), equation (22) becomes $k_n^*(p_n) = \arg\max_k (|\alpha_{k,n}|^2 p_n/(\sigma_{k,n}^2 \Gamma))^{w_k}$. On the contrary, in low-SNR regime the index of the optimal user is simply $k_n^* = \arg\max_k w_k |\alpha_{k,n}|^2/(\sigma_{k,n}^2 \Gamma)$, which is independent of p_n.

The above theorem proves the optimality, in terms of weighted sum of rates, of the exclusive subcarrier assignment, i.e. the OFDMA scheme, with an appropriate power allocation. OFDMA optimality was stated in [7] in the special case of equal weights. Another mathematically-similar result can be found in [10] where optimal resource allocation in code-division schemes is considered from an information-theoretical point of view. This is again a special case of (23) since it considers error-free data rates where the coding gap is equal to zero decibel ($\Gamma = 1$).

Now, we have to decide how to partition the overall power p_{tot} among the N subcarriers by resolving the second subproblem (21). This issue is discussed in the following section.

C. Optimal power allocation

From (16), the optimum subcarrier assignment $\mathbf{c_n^*}$ given by *Theorem 1* yields on the n^{th} subcarrier a weighted sum of rates equal to

$$r_n(p_n, \mathbf{c_n^*}(p_n)) = w_{k_n^*(p_n)} log_2 \left(1 + \frac{|\alpha_{k_n^*(p_n),n}|^2 p_n}{\sigma_{k_n^*(p_n),n}^2 \Gamma}\right),$$

where $k_n^*(p_n)$ is given by (22). Thus, the power allocation subproblem (21), consisting of maximizing the total weighted sum of rates, becomes equivalent to

$$r^* = \max_{p_1,..,p_N} \sum_{n=1}^{N} r_n(p_n, \mathbf{c}_\mathbf{n}^*(p_n)) \quad (24)$$

under the total power constraint (13). Unlike the case of equal weights in [7] where the power allocation step is resolved by traditional water-filling [12], the optimization problem (24) is more difficult. This is due to the dependency of the optimal scheduled user index in (22) on the amount of power p_n. In fact, all that (22) provides is the assertion that only one user is to be scheduled on each subcarrier. However, for a given user-scheduling configuration $(k_1, .., k_N)$ where k_n stands for the index of the scheduled user on the subcarrier n, we can prove, using *Lagrange multipliers method* [12], that the following sum

$$r = \sum_{n=1}^{N} w_{k_n} log_2 \left(1 + \frac{|\alpha_{k_n,n}|^2 p_n}{\sigma_{k_n,n}^2 \Gamma}\right) \quad (25)$$

is maximized by multilevel water-filling. This is true in particular for the optimal scheduling $(k_1^*, .., k_N^*)$. Thus, we can proceed by exhaustive search to find the optimal subcarrier allocation (or user scheduling) $(k_1^*, .., k_N^*)$. For each candidate scheduling $(k_1, .., k_N)$ of the K^N possible ones, we use the multilevel water-filling to calculate the corresponding powers and (25) to find the resulting weighted sum of rates. The optimal scheduling and power allocation correspond to the maximum weighted sum of rates. Such exhaustive search is certainly impractical because of its high complexity. A reduced-complexity search method can be derived from the results in [9] where a similar problem is handled in the Lagrange dual domain. Hereafter, we propose a simple suboptimal solution based on equal-power allocation.

D. Equal-power suboptimal solution

When the users undergo i.i.d. fadings on the different subcarriers, the powers allocated to the different subcarriers have obviously the same statistical average. Therefore, we expect that a subcarrier assignment based on equal powers $p_n = p_{tot}/N$ for all n, has a close-to-optimal performance in terms of average weighted sum of rates. This is confirmed by the simulation results presented later in Section IV. With equal powers, equation (22) becomes

$$\tilde{k}_n = \arg \max_{k=1,..,K} \left(1 + \frac{|\alpha_{k,n}|^2 \, p_{tot}}{\sigma_{k,n}^2 \Gamma \, N}\right)^{w_k} .$$

Thus, the suboptimal scheduling $(\tilde{k}_1, .., \tilde{k}_N)$ can be found with low complexity since no exhaustive search is involved. In the following section, we show how close this suboptimal solution is to the optimal one found by exhaustive search and multilevel water-filling. Then, we compare the suboptimal solution performance to a traditional static scheme where a fixed set of subcarriers is allocated to each user.

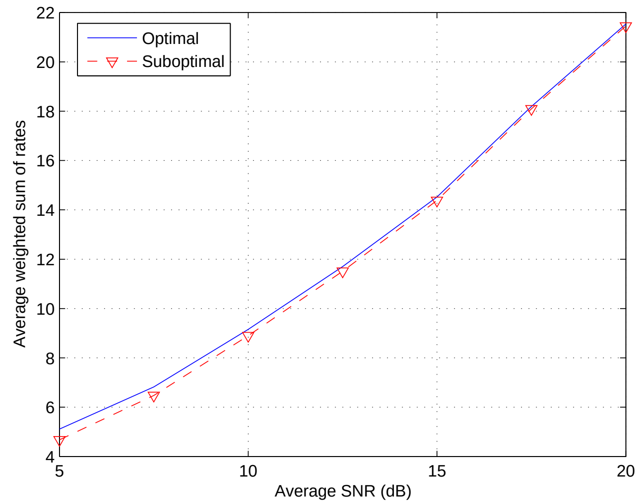

Fig. 2. Average weighted sum of rates versus average SNR for optimal and suboptimal solutions ($K = 2$ users, $N = 8$ subcarriers, $w_1 = 0.6$, $w_2 = 0.4$, $BER = 10^{-3}$, $P_{tot} = 1$, Number of channel realizations=1000).

IV. Simulation results

We consider an OFDM system with two users subject to i.i.d. Rayleigh fadings $|\alpha_{k,n}|$ with $E[|\alpha_{k,n}|^2] = 1$. We assume that the AWGN powers $\sigma_{k,n}^2$ are all equal to $\sigma^2 = N_0 \frac{B}{N}$ where N_0 is the noise power spectral density and B is the total bandwidth. We define an "average SNR" $\bar{\gamma}$ by

$$\bar{\gamma} = p_{tot}/(N_0 B) = p_{tot}/(N\sigma^2).$$

The value of N is limited to eight subcarriers in order to have bounded simulation time when exhaustive search is implemented. The BER value is $BER = 10^{-3}$ and all the data rates hereafter are normalized to the total bandwidth B.

In Fig. 2, we show the average weighted sum of rates versus the average SNR for the optimal solution found by exhaustive search and multilevel water-filling, as well as the suboptimal solution based on equal powers. The weighting factors are $(w_1, w_2) = (0.6, 0.4)$. As expected, there is no significant performance loss when the scheduling is decided with equal powers instead of multilevel water-filling-based powers.

Now, we want to compare the suboptimal adaptive subcarrier assignment to a static subcarrier assignment where a fixed set of subcarriers is allocated to each user. For equal weights $w_1 = w_2 = 0.5$, it seems natural to allocate $N/2$ subcarriers to each user in the static scheme. To make the comparison fair in the case of unequal weights, we propose to allocate N_1 and N_2 subcarriers to the first and to the second user respectively with $N_1/N_2 \simeq w_1/w_2$. This can be achieved by taking $N_1 = floor(w_1 N)$, $N_2 = N - N_1$ where $floor(x)$ is the closest integer to x. This subcarrier partitioning is better understood when Fig. 3 is considered. In Fig. 3, we plot the ratio of the average number of subcarriers allocated to user 1 to the total number of subcarriers N versus w_1 for both the optimal and the suboptimal solutions.

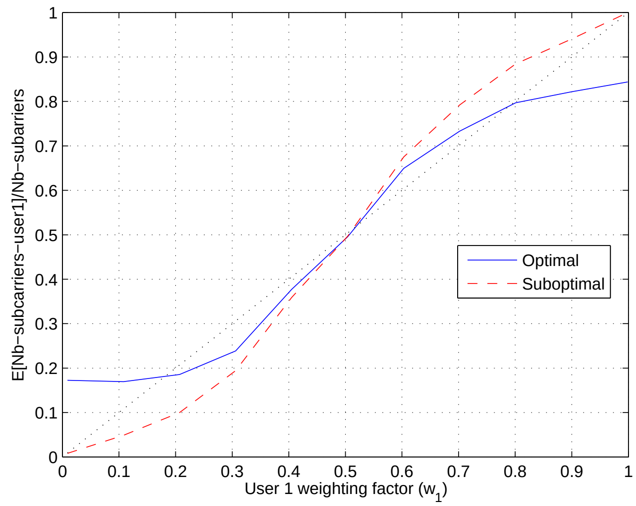

Fig. 3. Average number of subcarriers allocated to user 1 (normalized to the total number of subcarriers N) versus w_1 for optimal and suboptimal solutions ($K = 2$ users, $N = 8$ subcarriers, $BER = 10^{-3}$, $P_{tot} = 1$, Average SNR $\bar{\gamma} = 10dB$, Number of channel realizations=1000).

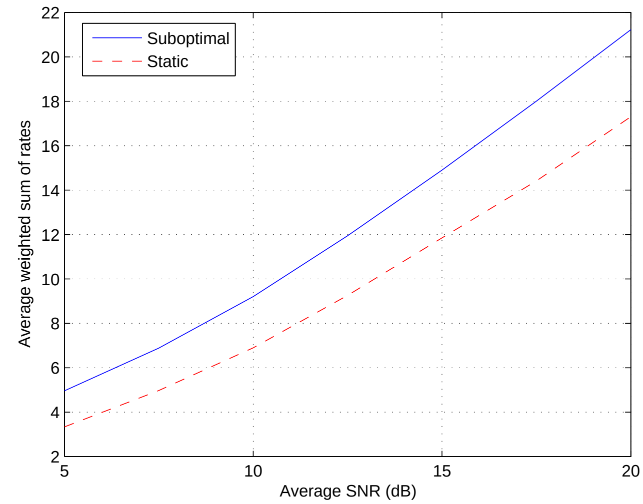

Fig. 4. Average weighted sum of rates versus average SNR for suboptimal and static allocation schemes ($K = 2$ users, $N = 8$ subcarriers, $w_1 = 0.6$, $w_2 = 0.4$, $BER = 10^{-3}$, $P_{tot} = 1$, Number of channel realizations=1000).

We see that this ratio can be approximated by w_1. This observation is independent of the average SNR value and of N. Consequently, it is fair to compare the weighted sum of rates obtained with the suboptimal solution to the weighted sum of rates corresponding to static subcarrier allocation where the first $floor(w_1 N)$ subcarriers are allocated to the first user and the remaining ones are allocated to the second user. This comparison is illustrated in Fig. 4 where the weighted sum of rates is plotted versus the average SNR for both the suboptimal and the static subcarrier allocation schemes. We see that a significant gain is achieved by adaptive subcarrier allocation even when this allocation is based on equal powers. This gain is due to the multiuser diversity exploited on each subcarrier when the scheduled user is dynamically chosen.

V. Conclusion

In this paper, we considered the problem of maximizing the weighted sum of rates on the downlink of a multiuser OFDM system under transmit peak power and target BER constraints. In our formulation, we allowed simultaneous sharing of each subcarrier by multiple users. We decomposed the resulting non-convex optimization problem into two tractable subproblems corresponding to subcarrier assignment and power allocation. Resolving the first subproblem showed that the optimal solution is an exclusive subcarrier assignment, or equivalently an OFDMA scheme. This means that only one user is allowed to transmit on each subcarrier. Regarding the second subproblem, the optimal power partitioning over the subcarriers can be obtained by a multilevel water-filling. Since the solutions to these two subproblems are inter-dependent, an exhaustive search is needed to find the optimal user scheduling. However, we proposed a suboptimal strategy to avoid the burden of exhaustive search. Our suboptimal solution is based on equal-power allocation and presents a close-to-optimal performance. Simulation results also showed that the proposed suboptimal strategy yields a significant gain in terms of average weighted sum of rates compared to a traditional static subcarrier allocation.

Future work will focus on the case of differentiated-QoS applications where a user-wise target BER has to be met.

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