OPTIMAL FREQUENCY-REUSE PARTITIONING FOR UBIQUITOUS COVERAGE IN CELLULAR SYSTEMS

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ABSTRACT

Frequency-reuse partitioning corresponds to allocating frequencies in a cellular system using multiple frequency-reuse factors simultaneously in each cell. Compared to single reuse-factor schemes, reuse partitioning allows an improved performance in terms of system capacity, outage probability or QoS fairness. In this paper we derive the optimal frequency-reuse partitioning that maximizes the capacity on the downlink of a cellular system. The main issue is to ensure a ubiquitous service in terms of user data rate assuming a total power constraint and a uniform user distribution.

1. INTRODUCTION

In cellular systems, frequency reuse is a key issue for optimizing the use of the scarce spectral resource. A reuse policy must cope with the problem of inter-cell interferences which becomes critical for edge users. Several approaches for *Co-Channel Interference* (CCI) mitigation do exist. They can be classified into three main categories: CCI cancellation, CCI randomization and CCI coordination and avoidance. CCI cancellation methods [2] are performed at the receiver side based on multiuser detection and successive interference substraction or on multi-antenna spatial separation techniques [3]. Such methods suffer from hardware implementation constraints. Interference randomization [1] is based on random scrambling, spectrum spreading or frequency hopping. These techniques transform narrow-band interfering signals into a less aggressive wide-band noise. CCI coordination and avoidance [4] remains the most attractive family of interference mitigation methods. These methods are based on cooperation between the transmitters (base stations on the downlink) in order to adopt an optimal common strategy for available resources' (time, frequency, power, code and/or space) utilization. Frequency planning and user scheduling are two examples of such strategies. *Frequency-Reuse Partitioning* belongs also to this family. It consists in dividing the available bandwidth into two or more sub-bands with different frequency-reuse factors (FRF) (See [9] and references therein). Frequency-reuse partitioning was first introduced in [5] where it was reported that a significant capacity gain can be achieved by using two reuse factors instead of a single one. Finding the optimal frequency-reuse partitioning becomes more challenging when some QoS fairness constraints are imposed. This problem was considered in [6,7] where the authors tried to guarantee a uniform call-blocking probability in a non-sectorized cellular system. In [8] a 3-sector model was assumed and the frequency-reuse factor of each subcarrier was adapted dynamically in order to minimize a kind of minimum-rate violation ratio. Finally, frequency-reuse partitioning was coupled in [9] with power control in order to minimize the rate-outage probability in the entire system.

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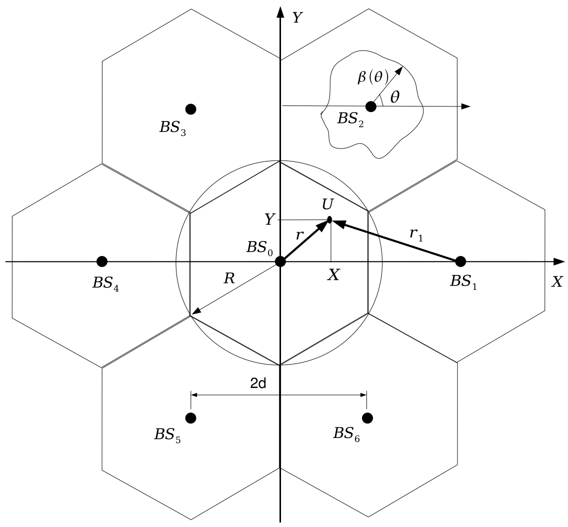

Figure 1: Cell cluster and reuse-partitioning boundary $\beta(\theta)$.

In this paper, we are interested in maximizing the cell capacity on the downlink using resuse-partitioning under the constraint of a ubiquitous service coverage. This means that the same data rate is guaranteed whatever the user position is inside the cell. We tackle this problem analytically assuming that the channel effect is reduced to pathloss with arbitrary exponent value. This allows us to get a first evaluation of the achievable performance. The effect of more realistic propagation models with random shadowing and multipath fading is left for further investigation.

2. SYSTEM MODEL

Consider a cellular system with hexagonal cells of radius R as shown in Figure 1. In this figure is depicted a cluster of seven cells composed of a central cell around base station BS_0 in addition to its six direct neighbors BS_1,...,BS_6. We suppose that the same cluster is repeated infinitely in all directions. Each BS uses an omnidirectional antenna. The distance between any two adjacent BS's is $2d$ with $d = 0.5\sqrt{3}R$. We assume a constant and continuous user density (number of users by unit area) everywhere. A total transmit power P_{tot} is available to each BS. Whether a given BS occupies the whole bandwidth B_{tot} or not, it transmits with a constant power spectral density $p = P_{tot}/B_{tot}$. The channel power gain between two points separated by a distance r is restricted to the pathloss $G(r)$. We use the exponent pathloss model [10] given by $G(r) = G_0/r^\alpha$ where the pathloss exponent α depends on the terrain nature and on antenna heights. The constant G_0 is given by $G_0 = (c/(4\pi f))^2$ where f is the center frequency and c is the speed of light.

With a reuse factor FRF=1, a user in $U(X,Y)$ at distance $r = \sqrt{X^2+Y^2}$ from BS_0 (see Figure 1) receives on a given frequency a useful signal from BS_0 in addition to additive white Gaussian noise and interferences from other BS's transmitting on the same frequency. Due to fast signal decay with distance, we only consider the interferences caused by the BS's of the direct neighboring cells BS_1,...,BS_6. Thus, six interfering terms appear in the expression of the *Signal-to-Interference-plus-Noise Ratio* (SINR) at point (X,Y) as follows

$$\Gamma(X,Y) = \frac{p\ G_0/r^\alpha}{N_0 + \sum_{k=1}^{k=6} p\ G_0/r_k^\alpha} \tag{1}$$

where r_k is the norm of the vector $\overrightarrow{BS_k\,U}$. Let (x,y) be the coordinates normalized to R, i.e. $(x,y) = (X/R, Y/R)$. Thus, the normalized coordinates of the interfering BS's are $BS_1(\sqrt{3},0)$, $BS_2(\frac{\sqrt{3}}{2},\frac{3}{2})$, $BS_3(-\frac{\sqrt{3}}{2},\frac{3}{2})$, $BS_4(-\sqrt{3},0)$, $BS_5(-\frac{\sqrt{3}}{2},-\frac{3}{2})$, $BS_6(\frac{\sqrt{3}}{2},-\frac{3}{2})$. It follows from (1) that the SINR expression in normalized coordinates becomes

$$\gamma(x,y) = \frac{\Gamma_e}{(x^2+y^2)^{\alpha/2}\,[1+\Gamma_e\, S(x,y)]} \tag{2}$$

where Γ_e is *the edge SNR* (without considering interferences) defined by

$$\Gamma_e = \frac{p\ G_0}{N_0 R^\alpha} \tag{3}$$

and $S(x,y)$ is given by

$$\begin{aligned} S(x,y) = &\left[\left(x-\sqrt{3}\right)^2+y^2\right]^{\frac{-\alpha}{2}} + \left[\left(x+\sqrt{3}\right)^2+y^2\right]^{\frac{-\alpha}{2}} \\ &+\left[\left(x-\frac{\sqrt{3}}{2}\right)^2+\left(y-\frac{3}{2}\right)^2\right]^{\frac{-\alpha}{2}} \\ &+\left[\left(x+\frac{\sqrt{3}}{2}\right)^2+\left(y-\frac{3}{2}\right)^2\right]^{\frac{-\alpha}{2}} \\ &+\left[\left(x+\frac{\sqrt{3}}{2}\right)^2+\left(y+\frac{3}{2}\right)^2\right]^{\frac{-\alpha}{2}} \\ &+\left[\left(x-\frac{\sqrt{3}}{2}\right)^2+\left(y+\frac{3}{2}\right)^2\right]^{\frac{-\alpha}{2}}. \end{aligned} \tag{4}$$

In the next section the problem of finding the optimal frequency-reuse partitioning is formulated.

3. DEFINITIONS AND PROBLEM FORMULATION

The idea behind the frequency-reuse partitioning is the following. Since the CCI becomes critical for boundary users sharing the same frequency band, the solution is to prevent frequency sharing by adjacent cells for such users. This means that adjacent cells must cooperate in order to select a non-overlapping sub-bands that will be allocated to boundary users with FRF>1. On the other hand, users that seem "isolated" from CCI thanks to their "deep" location (close to the serving BS) in their respective cells are allowed to share simultaneously the remaining frequencies. This defines a zone around the BS of each cell where users are allocated a common frequency-band with FRF=1. We call this inner zone *the Full-Reuse Zone* (FR-Zone) while the rest of the cell forms the *Partial-Reuse Zone* (PR-Zone). The boundary between these two zones is referred to as *the partitioning boundary*. It can be represented in polar coordinates by the function $r = \beta(\theta)$ with the origin at the center of the considered cell. This boundary is plotted in Figure 1 for cell BS_2. Notice that the SINR in the partial-reuse zone is reduced to a SNR (no interference) so that equation (2) is replaced by

$$\gamma_\beta(x,y) = \begin{cases} \frac{\Gamma_e}{(x^2+y^2)^{\alpha/2}[1+\Gamma_e\, S(x,y)]}, & (x,y) \in \text{FR-Zone} \\ \frac{\Gamma_e}{(x^2+y^2)^{\alpha/2}}, & (x,y) \in \text{PR-Zone} \end{cases} \tag{5}$$

The subscript β was added to the SINR symbol in (5) to emphasize its dependency on the function $\beta(\theta)$.

As for the central cell in Figure 1, each surrounding cell is in its turn located at the center of a new cluster and suffers from the same interference profile. Thus, we can assume that in each cell the partitioning boundary is the same.

Let $b(x,y)$ be the amount of bandwidth allocated by BS_0 to users inside a differential zone around (x,y). We call $b(x,y)$ *the bandwidth allocation density*. This assumes a continuous bandwidth allocation (infinite number of subcarriers in the OFDMA case). Thus, a user at (x,y) achieves a *capacity density* $c_\beta(x,y)$ in bps/m^2 given by

$$c_\beta(x,y) = b(x,y)\log_2(1+\gamma_\beta(x,y)) \tag{6}$$

with $\gamma_\beta(x,y)$ defined in (5). Obviously, this capacity density depends on the chosen partitioning boundary described by $\beta(\theta)$. The capacity achieved in a cell can be obtained by integrating (6) over the cell area.

Our aim is to offer the best ubiquitous QoS coverage over the whole cell. With a uniform user density, this means that $c_\beta(x,y)$ is equal to a constant C_β that we want to maximize with respect to the partitioning boundary $\beta(\theta)$ and to the bandwidth allocation scheme $b(x,y)$. This is equivalent to maximizing the capacity per cell which is given by the product $C_\beta\ \times$ cell area. Thus, the considered optimization problem can be formulated as follows

$$\begin{aligned} &\max_{\beta,\,b}\ C_\beta \\ &\text{subject to}\ \iint_{Cell} b(x,y)\mathrm{d}x\,\mathrm{d}y \le B_{tot}. \end{aligned} \tag{7}$$

Here the inequality corresponds to the total-bandwidth constraint. This is an unusual problem since the optimization variables are the unknown functions $\beta(\theta)$ and $b(x,y)$. In the following section a solution to (7) is proposed under a given frequency-reuse pattern.

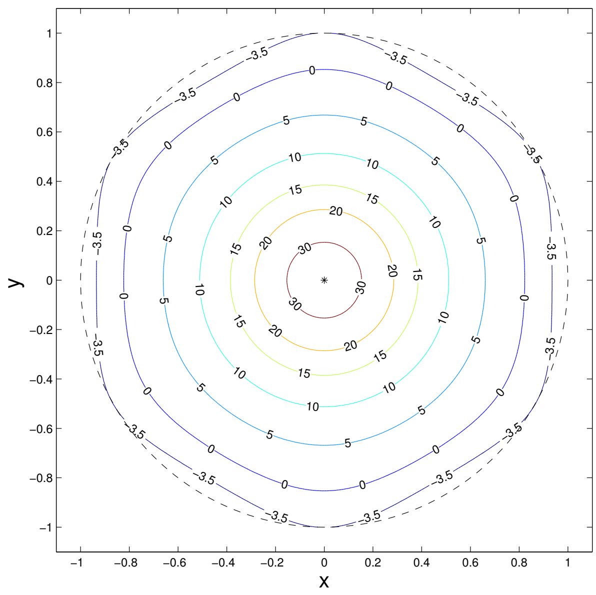

Figure 2: Constant-SINR contours (Contours values are in dB's, x-y coordinates are normalized to the cell radius R).

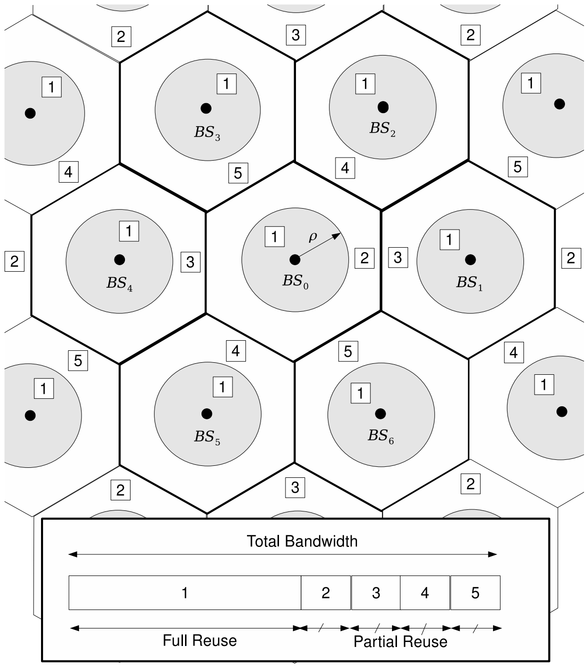

Figure 3: Proposed frequency-reuse pattern and total bandwidth partition.

4. OPTIMAL REUSE PARTITIONING

We start by simplifying the considered problem (7) by making some assumptions on the frequency-reuse partitioning boundary. Since classifying a given point into full or partial reuse scheme depends on the CCI amount or, equivalently, on the SINR value at that point, it is interesting to take a look on constant-SINR contours assuming full reuse everywhere. The SINR is given in equation (2). An example of these contours is depicted in Figure 2. It corresponds to the parameter setting specified farther in Section 5. Unsurprisingly, these contours are approximately circular near the BS where the noise is dominant compared to the CCI. For low SINR values, the corresponding contours tend to a quasi-hexagonal form. In our analysis, we assume that the frequency-reuse partitioning boundary corresponds to a circle so that $\beta(\theta) = \rho$ for all θ. So, by considering a circular cell boundary of radius R and by taking $x = r,\ y = 0$ (polar coordinates with $\theta = 0$), the SINR expression (5) can be simplified to

$$\gamma_\rho(r) = \begin{cases} \frac{\Gamma_e}{r^\alpha[1+\Gamma_e\, s(r)]} & \text{if } \ 0 < r \le \rho, \\ \frac{\Gamma_e}{r^\alpha} & \text{if } \ \rho < r \le 1, \end{cases} \tag{8}$$

where $s(r) = S(r,0)$. From (4) we get

$$s(r) = \left(r - \sqrt{3}\right)^{-\alpha} + \left(r + \sqrt{3}\right)^{-\alpha}$$
$$+2\left[\left(r - \frac{\sqrt{3}}{2}\right)^2 + \frac{9}{4}\right]^{\frac{-\alpha}{2}} + 2\left[\left(r + \frac{\sqrt{3}}{2}\right)^2 + \frac{9}{4}\right]^{\frac{-\alpha}{2}} \tag{9}$$

In (8), the SINR is parametrized by the partitioning boundary radius ρ. Note that both r and ρ are normalized to R. The capacity density (6) can be, in its turn, written as follows

$$c_\rho(r) = b(r)\log_2(1+\gamma_\rho(r)). \tag{10}$$

Having a constant capacity density $c_\rho(r) = C_\rho$ allows us to derive from (10) the *bandwidth allocation density* as follows

$$b(r) = \frac{C_\rho}{\log_2(1+\gamma_\rho(r))}. \tag{11}$$

Note that the subscript β of the constant capacity density has been replaced by the radius ρ. We deduce that the bandwidth allocated to the FR-Zone is

$$B_{FR} = 2\pi\, C_\rho \int_0^\rho \frac{r\,\mathrm{d}r}{\log_2(1+\gamma_\rho(r))}. \tag{12}$$

Here we have a simple integral over the FR-Zone since the Cartesian differential area $(\mathrm{d}x\,\mathrm{d}y)$ is replaced in polar coordinates by $(2\pi r\,\mathrm{d}r)$. One must notice that the remaining bandwidth $B_{tot} - B_{FR}$ has to be partitioned among several cells in a way that depends on the adopted *frequency-reuse pattern*. We propose the pattern depicted in Figure 3 where the partial-reuse bandwidth $B_{tot} - B_{FR}$ is divided into four equal sub-bands. Note that the partitioning boundaries are represented by circles according to our assumptions above. This reuse pattern yields an FRF=4 in the PR-Zone. The minimum reuse distance is equal to $4d = 2\sqrt{3}R$. This allows us to neglect any CCI in the partial-reuse sub-bands.

Now, based on (10), we define the *spectral efficiency density* $(bps/Hz/m^2)$ by

$$e(r) = \frac{c_\rho(r)}{b(r)} = \log_2(1+\gamma_\rho(r)). \tag{13}$$

Table 1: Parameter setting.

Center frequency f	3.5	GHz
Total bandwidth B_{tot}	20	MHz
Maximum total power	10	W
Noise spectral density N_0	-174	dBm/Hz
Pathloss exponent α	3.6	-
Target cell radius R	100	m

The quantity $e(r)$ has a crucial role in the following proposition on the optimal frequency-reuse partitioning.

Proposition 1

The cell capacity in the described system is maximized for a frequency-reuse partitioning radius at which the spectral efficiency density without CCI is equal to the spectral efficiency density with CCI times the frequency-reuse factor of the partial-reuse zone.
In other words, if ρ^* denotes the optimal frequency-reuse partitioning radius, then, using (13) we have

$$\log_2\left[1+\frac{\Gamma_e}{(\rho^*)^\alpha}\right] = 4\log_2\left[1+\frac{\Gamma_e}{(\rho^*)^\alpha\left[1+\Gamma_e\, s(\rho^*)\right]}\right]. \quad (14)$$

The proof is detailed in the Appendix.

Due to the complex form of (14), it is difficult to derive an analytic expression for ρ^*. Note however that the solution depends only on two parameters: the edge SNR Γ_e and the pathloss exponent α. In the next section, we resolve this equation numerically. Once ρ^* is found, the optimal bandwidth allocation density can be obtained using (11) with $\rho = \rho^*$. Then, the full-reuse bandwidth B_{FR} can be found from (12).

5. NUMERICAL RESULTS

Consider the parameter setting of Table 1. In Figure 4 is plotted the optimal frequency-reuse partitioning radius ρ^* versus the total power P_{tot}. This curve represents the numerical solution of equation (14).

In Figure 5 we show the variation of the maximum achievable capacity per cell versus the partitioning radius for different values of P_{tot}. Cell capacity is normalized to the total bandwidth B_{tot}. When $\rho = 0$, the whole band B_{tot} is divided into four sub-bands with FRF=4. In this case, all frequencies are interference-free so that increasing the total power enhances the cell capacity. When ρ increases, the BS's start to share some sub-band with FRF=1. This increases the cell capacity up to a maximum at $\rho = \rho^*$. Beyond the optimal radius ρ^* the capacity decreases and the system tends to a full-reuse scheme in which all frequencies are subject to CCI. For $\rho^* = 1$, increasing the total power per cell has a marginal effect on cell capacity since interferences increase also.

6. CONCLUSION

We considered the problem of maximizing the capacity on the downlink of a cellular system using frequency-reuse par-

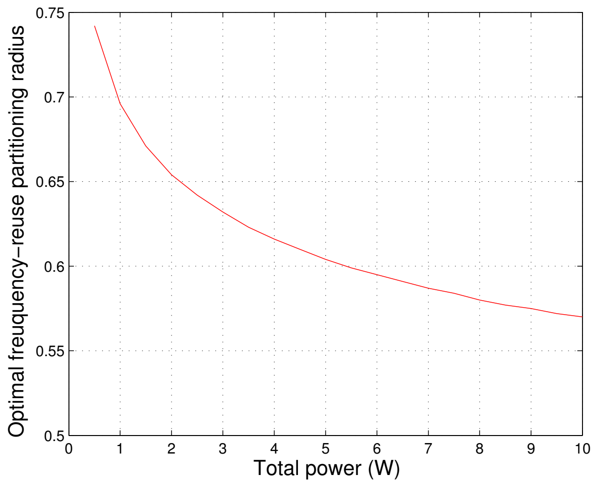

Figure 4: Optimal reuse-partition radius versus total power.

titioning under the constraint of a ubiquitous QoS coverage in terms of user data rate. The problem was formulated assuming a continuous and uniform user distribution as well as a continuous bandwidth allocation. A frequency-reuse pattern was proposed and a method for optimizing the bandwidth sharing was described. Numerical results illustrated the cell capacity variation and confirmed the existence of an optimal solution.
Future work will investigate the effect of an OFDMA subcarrier-based allocation with a frequency-selective random channel.

Appendix

Here we provide the proof of *Proposition 1* formulated mathematically in (14). According to the frequency-reuse pattern proposed in Figure 3, we can write

$$2\pi\int_0^\rho b(r)r\,\mathrm{d}r + 4\times 2\pi\int_\rho^1 b(r)r\,\mathrm{d}r = B_{tot}. \quad (15)$$

From (11) and (15) we get

$$\int_0^\rho \frac{r\,\mathrm{d}r}{\log_2\left(1+\frac{\Gamma_e}{r^\alpha[1+\Gamma_e\, s(r)]}\right)} + \int_\rho^1 \frac{4r\,\mathrm{d}r}{\log_2(1+\frac{\Gamma_e}{r^\alpha})} = \frac{B_{tot}}{2\pi C_\rho} \quad (16)$$

Thus, the optimization problem (7) can be re-formulated as follows

$$\max_\rho C_\rho \quad \text{subject to (16)}.$$

From (16) we obtain

$$C_\rho = \frac{B_{tot}}{2\pi I(\rho)} \quad (17)$$

with

$$I(\rho) = \int_0^\rho \frac{r\,\mathrm{d}r}{\log_2\left(1+\frac{\Gamma_e}{r^\alpha[1+\Gamma_e\, s(r)]}\right)} + \int_\rho^1 \frac{4r\,\mathrm{d}r}{\log_2(1+\frac{\Gamma_e}{r^\alpha})}. \quad (18)$$

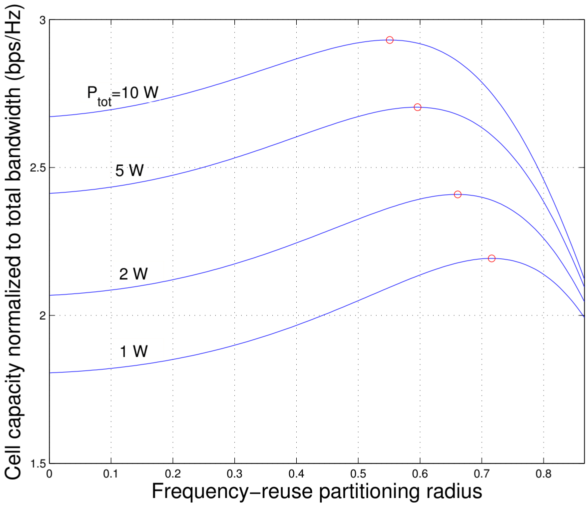

Figure 5: Cell capacity variation versus partition radius for different values of total power.

According to (17), maximizing C_ρ is equivalent to minimizing the function $I(\rho)$. Note that

$$\frac{\mathrm{d}}{\mathrm{d}\rho} I(\rho) = \frac{\rho}{\log_2\left(1 + \frac{\Gamma_e}{\rho^\alpha [1+\Gamma_e\, s(\rho)]}\right)} - \frac{4\rho}{\log_2(1+\frac{\Gamma_e}{\rho^\alpha})}. \quad (19)$$

One can check that the second derivative of $I(\rho)$ is positive for $\rho \in [0,1]$ so that $I(\rho)$ reaches its global minimum when its first derivative (19) is null. Thus, from $\mathrm{d}I(\rho)/\mathrm{d}\rho = 0$ we retrieve equation (14) that gives the optimal frequency-reuse partitioning radius ρ^*.

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